A STUDY OF TRANSIENT EXCITATIONS USEFUL FOR DETERMINING VEHICLE CHARACTERISTICS

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by

E. HESTAD

Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

Progress Report

A STUDY OF TRANSIENT EXCITATIONS USEFUL FOR DETERMINING VEHICLE CHARACTERISTICS

TO: K. B. Woods, Director
Joint Highway Research Project

July 22, 1964

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 6-20-6
Project: C-36-52F

The attached report "A Study of the Transient Excitations Useful for Determining Vehicle Characterisitcs" by Mr. Erling Hestad, Graduate Assistant on our staff, has been prepared as a Progress Report on the "Stresses and Deflections" cooperative project. Mr. Hestad also used the research as the basis for his thesis for the MSME degree and conducted the research under the guidance of Professor B. E. Quinn.

The report is concerned with obtaining certain vehicle characteristics related to the dynamic force of the vehicle on the highway from transient excitations. The research indicated that acceptable vehicle characteristics can be obtained by using pulse and step inputs.

The report is submitted for the record and will also be transmitted to the Highway Commission and the Bureau for review and comment.

Respectfully submitted,

H. L. Michael

H. L. Michael, Secretary

HLM:bc

Attachment

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Progress Report

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By

Erling Hestad
Graduate Assistant

Joint Highway Research Project

Project: C-36-52F

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Special appreciation is expressed to Dr. Bayard E. Quinn, Professor of Mechanical Engineering, who as project director and major professor provided the guidance, advice and aid which led to the results of this investigation. Particular appreciation is also expressed for his guidance and help in the preparation of this thesis.

The help of fellow members of the Vehicle Dynamics Research Group, Messrs. Clement C. Wilson, Gary Kibbee and Jack Zable, was invaluable.

The careful work done by Messrs. Jon Beeson and Robert Brossart in the preparation of illustrations for this thesis has been greatly appreciated.

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ABSTRACT

Hestad, Erling. M.S.M.E., Purdue University, August 1964. A Study of Transient Excitations Useful for Determining Vehicle Characteristics. Major Professor: Bayard E. Quinn.

The vertical force from a tire of a moving vehicle on a highway can be described as a superposition of a constant static wheel load and a fluctuating dynamic load, F, introduced by unevenness of the highway pavement. The dynamic force F is determined by measuring the fluctuating pressure P in the tire. If it is desired to predict the dynamic force from the highway profile, the relationship between F and the input displacement X at the tread of the tire must be known.

The possibility of determining the vehicle characteristics or calibration relationships F/X and F/P by using a steady state excitation or a transient displacement input to the wheel of the car was considered.

Particular attention was devoted to the possibility of obtaining the dynamic characteristics F/P and F/X by using transient excitations. By using the Fourier transformation, the displacement, force and pressure relation-

ships obtained in the time domain were transformed into the frequency domain where the significant characteristics could be studied.

Since step and pulse displacement inputs have been the commonly used transient excitation inputs, an attempt was made to show how these inputs could be improved to give a better frequency spectrum for exciting the vehicle. The possibility of using a transient multiple pulse displacement input was considered.

The experimental testing was performed on a passenger car, but the results obtained are also applicable to heavier vehicles.

The investigation indicates that acceptable vehicle characteristics can be obtained by using the pulse and step displacement inputs. However, a greater degree of accuracy will require the use of a driving mechanism to produce special types of displacement excitations, such as a pulse displacement with a high amplitude and short time duration, or a multiple pulse. These transient excitations will be needed especially if an accurate determination of the vehicle characteristics in the higher frequency range is desired.

CHAPTER 1

INTRODUCTION

To deal effectively with a physical system it is often necessary to determine the dynamic characteristics of the system. Information about the dynamic characteristics can be acquired by determining the relationship between "input" and "output" quantities of the system. The terms input and output take their meaning from the physical situation under consideration. To give an example; a voltage pulse can be used as the input excitation to an electric circuit, and the resulting voltage pulse over the output terminals can be called the resulting voltage output of the system.

If a physical system is very complicated so that a mathematical analysis is difficult, an experimental procedure to predict the system characteristics is desired. A commonly used technique is to apply a steady state sinusoidal input to the system and to measure the steady state output. If the input frequencies can be varied to cover a desired frequency band, this method will then give the system characteristics as a function of frequency for this frequency band.

However, interest in another method, which can be called a transient excitation method, has been steadily increasing in recent years. The transient excitation method is an indirect frequency response method, where system characteristics are acquired from the time histories of transient inputs and the resulting responses of the physical system. Since dynamic systems are frequency dependent, a comparison of the input and output in the frequency domain is desired. This can be accomplished by the Fourier transform technique to "convert" the input and output time histories to the frequency domain. If a digital computer is available, the frequency domain description of the input and output of the system can be obtained by employing a numerical integration technique. The dynamic characteristic or calibration function, output divided by input, as a function of frequency can then be obtained.

Transient excitation has been used in many areas to determine the dynamic behavior of physical systems. For example, this method has been shown to be especially applicable for determining dynamic characteristics of airplanes. Muzzey and Kidd[1] used both steady state sinusoidal inputs and transient excitation inputs to move appropriate control surfaces of an airplane. The airplane motion and the control motion were recorded and an analysis of the dynamic characteristic of the airplane was then performed. In a comparison of these two methods they conclude that in

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1. Superscripts refer to references listed in the Bibliography.

general the transient excitation method is superior to a steady state sinusoidal excitation. Smith and Triplett[2] discuss the problem of obtaining dynamic characteristics of an airplane by using different transient inputs.

The transient excitation method for determining the dynamic characteristics can be expected to give valid results if the system is behaving linearly. By linear behavior of the system is meant that the ratio of the output to the input at any specific frequency is constant for variation of input amplitude. Can the dynamic characteristics of a nonlinear system be determined by using a transient excitation? If the mathematically exact dynamic characteristics are desired, the answer must be no. However, if the nonlinearities in the system have a small effect on the behavior of the system, the transient excitation method can give important information from an engineer's viewpoint. If a nonlinear mathematical analysis is difficult or impossible to perform, then the transient excitation method or a steady state sinusoidal method may give the only available valid information about the dynamic characteristics of the system.

Can the transient calibration method be used to determine the dynamic characteristics of a vehicle? Before this question can be answered, it is necessary to define the dynamic characteristics of a vehicle and to explain why the determination of these dynamic characteristics is

important.

The vertical force from a tire of a moving vehicle on a highway can be described as a superposition of a static wheel load and a dynamic tire force. The static wheel load is the vertical force from a tire of a motionless vehicle on a level highway. When the vehicle is moving, unevenness of the pavement will introduce dynamic forces which are largely vertical in direction. The vertical dynamic force from a tire on a moving vehicle will be called the dynamic tire force, or just the dynamic force. The determination of the magnitude and the nature of the dynamic force is very important, since the dynamic force, in addition to the static wheel load, may result in high contact stresses and thus cause rapid deterioration of the pavement.

Several methods have been investigated for determining the dynamic force. On this project, force measurements are made by recording the variation in air pressure occuring in a tire as the vehicle travels along a highway. A schematic drawing of the system for measuring the variation in tire pressure is shown in Figure 1. Before making measurements, the valve to the reference tank is opened to establish the static air pressure in all parts of the pressure measuring system. Then the valve is closed, thus subjecting side A of the differential pressure transducer to the fluctuation in tire pressure when a calibration or

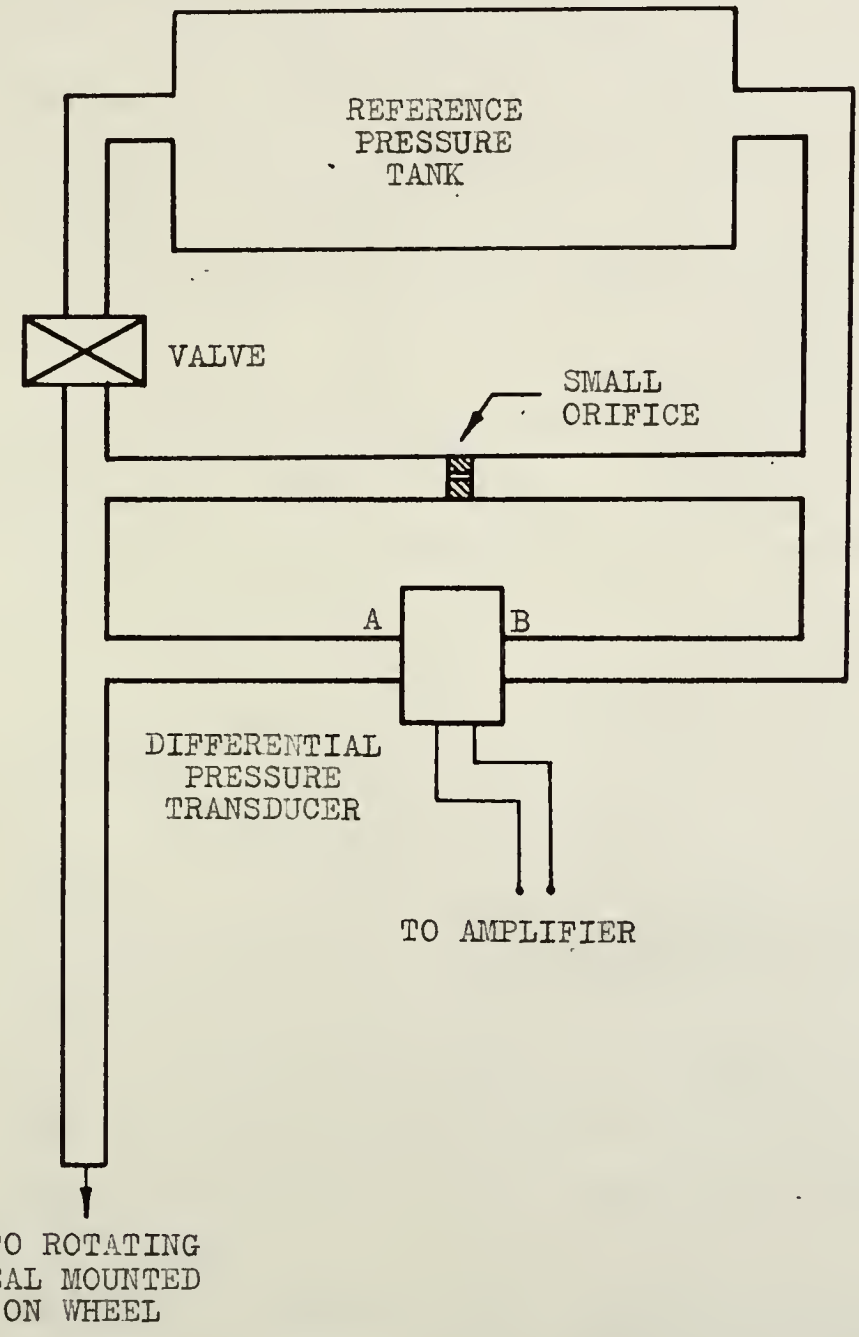


FIGURE 1. SCHEMATIC DIAGRAM OF SYSTEM FOR MEASURING CHANGE IN TIRE AIR PRESSURE

road test is performed, while side B is subjected to the original tire pressure. The pressure transducer measures the difference in pressure between side A and B, and the pressure difference information is transmitted to an amplifier and recorded by an oscillograph. The orifice is used to eliminate slow changes in tire pressure caused by heating or cooling of the tire when road tests are performed.

An illustration of the dynamic force and the change in tire pressure is given in Figure 2. The tire pressure is indicated by "P" and the associated change in the force of the tire on the highway is called "F".

If it is desired to predict the dynamic force on a highway from the highway profile, the relationship between F and the input displacement "X" to a tire of the vehicle must be known. If it is desired to determine the dynamic tire force from the tire pressure measurements, then the relationship between F and the tire pressure P must be available. It is then clear that two calibration ratios, F/X and F/P, are needed when the dynamic force from a moving vehicle on a highway is to be predicted or experimentally determined.

What kind of transient input excitations should be used to get valid information about the dynamic characteristics of a physical system? Before an attempt is made to answer this question it is important to realize that the

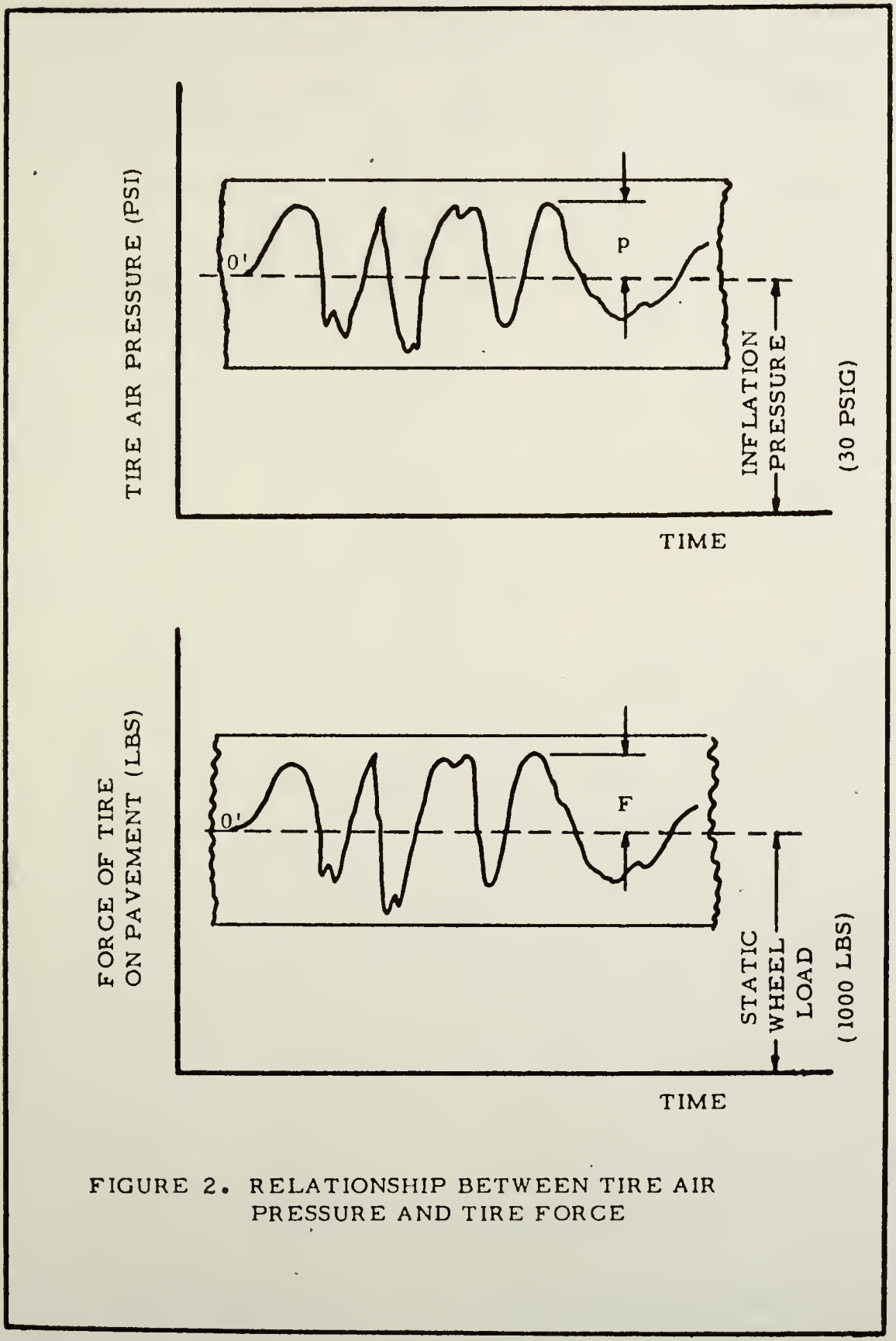


FIGURE 2. RELATIONSHIP BETWEEN TIRE AIR PRESSURE AND TIRE FORCE

input excitations will depend on the physical situation at hand. While an electrical circuit may easily be exposed to a square or a triangular wave voltage input, a physical realizable displacement input to the wheel of a heavy truck may consist of a modified step displacement. In work previously done, the transient displacement inputs for passenger vehicles have been a step or a pulse displacement input. The question thus naturally arises: Will these step or drop displacements give an accurate determination of the dynamic characteristics of the truck? If not, what kind of displacement inputs should be used? Are these displacement inputs physical realizable?

The literature contains some information that can be valuable, when deciding what kind of displacement inputs should be used to accurately determine the dynamic characteristics of vehicles. Dreifke[3] determined the dynamic characteristics of several well defined dynamic systems by simulating these systems on an analog computer and imposing different inputs to the systems. The experimental and the analytically calculated frequency responses could then be compared. In this manner, it was possible to relate the accuracy of the frequency response to the shape and type of the transient input.

The importance of the frequency description of input pulses is given in the following quotation:[3]

"It appears beyond doubt that the relative

> spectral content of the input pulse is of extreme importance and will always serve as a reliable basis for choosing the input shape or width. In contrast, there appears to be no alternate simple statement regarding long or short, big or little, this or that shape pulses to cover the range of possible experimental requirements."

A very long step input was shown to give reliable low frequency information for the most part, but this type of input, however, gave poor results at higher frequencies. This might be explained by the fact that the relative frequency spectra of a long pulse is high in the lower frequency range, but is decreasing very rapidly in the higher frequency range.

Smith and Triplett[2] point out that if a pulse is required to give good results in the higher frequency range the time base of the pulse should be decreased and the amplitude of the pulse should be increased. If highly accurate data are needed over a limited frequency band, a multiple pulse can be used.

To determine the desired dynamic characteristics of a vehicle, F/X and F/P, a good calibration procedure is needed to get valid results. Thompson[4] showed by using a steady state vibrator that the F/X ratio had a nonlinear characteristic. This means that the F/X ratio for any

specific frequency changed when the input amplitude was changed. However, it was believed that in spite of the nonlinear behavior, the steady state sinusoidal displacement input gave valuable information about the F/X ratio.

Later Hamilton[5] compared the F/X calibration ratio obtained by using both a steady state and transient displacement inputs (step, pulse) to the wheel of a passenger car. Hamilton concluded that the two methods gave approximately the same results over the entire frequency region under investigation. However, the transient excitation method appeared to give more consistent results in the lower frequency region. Hamilton maintains that since the equipment needed for performing transient excitation calibration is very easy to build and operate, this method is superior to steady state sinusoidal excitation of vehicles.

McLemore[6] built a transient excitation device to be used for trucks. Although this device is mainly designed for producing a step displacement input, provisions were made for applying a prescribed displacement to the wheel of a truck.

In the work previously done using transient displacement excitations to determine the dynamic characteristics of vehicles, the emphasis has been placed on building equipment for simple transient excitation tests. The question that now needs to be answered is: Will the pre-

sent transient excitation tests give an accurate determination of the desired vehicle characteristics? If not, what kind of transient excitations should be used to give a more reliable determination of the F/P and F/X characeristics.

CHAPTER 2

COMPARISON OF DIFFERENT TYPES OF CALIBRATION PROCEDURES TO DETERMINE THE VEHICLE CHARACTERISTICS

In general a physical system may be calibrated either statically or dynamically. An example of a static calibration is to slowly apply an increasing force to the tire by loading the vehicle, recording the force F and the corresponding pressure increase P in the tire. By computing the ratio F/P a calibration factor can be obtained. However, when a static test is performed one cannot identify the dynamic characteristics of the system, such as natural frequencies or the damping present in the system.

A calibration factor F/P or F/X which is independent of frequency will be advantageous and thus may be called an ideal characteristic. Quinn and Wilson[7] showed an ideal characteristic of a constant ratio of dynamic force to pressure along with an undesirable characteristic that is frequency dependent. An illustration is given in Figure 3.

With an ideal characteristic it is possible to use a time domain history, such as a pressure fluctuation record, and determine the dynamic force that will exist on the highway at any selected time. The undesirable charac-

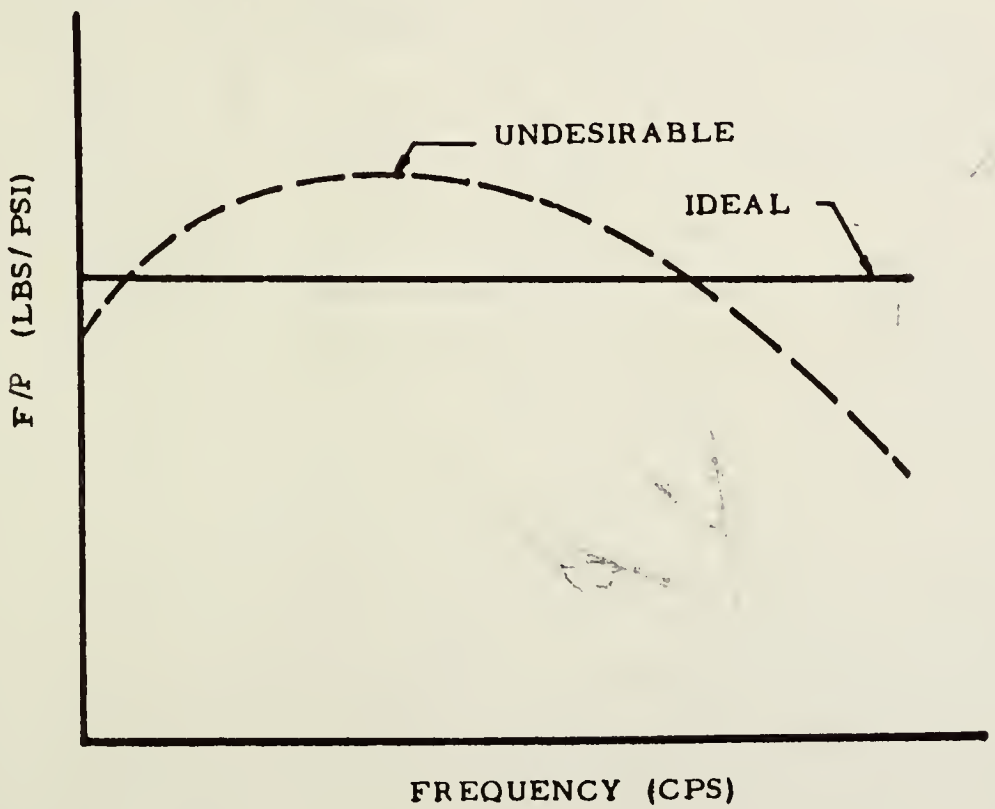


FIGURE 3. CALIBRATION CURVES FOR PRESSURE MEASURING SYSTEM

teristic, however, requires transformation of time domain records over to the frequency domain in order to get valid results.

When a dynamic calibration is performed, the ratio of tire force to tire pressure, F/P, will be found to vary with frequency. Dealing with a dynamic system it is also clear that the ratio F/X will be frequency dependent. Thus it is obvious that a dynamic or a frequency response calibration must be performed to obtain the actual F/P and F/X characteristics or calibration functions.

A straightforward mathematical analysis may be used to get the desired calibration functions in the frequency domain if the mathematical model of the system is known and the system parameters are well defined. However, a representative mathematical model for a vehicle will turn out to be very complex. Also the system parameters are not available for a wide variety of vehicles, and the experimental determination of the parameters may turn out to be a tedious task.

The possibility remains to use an experimental dynamic calibration procedure. In general, there are three experimental methods available for obtaining a relationship of F/X and F/P versus frequency for a vehicle. These are:

1. Steady state sinusoidal excitation
2. Transient tests or aperiodic excitation
3. Random excitation.

De Vries[8] designed a steady state sinusoidal vibrator to be used for determining suspension characteristics of a passenger car in which the vehicle wheel was positioned on a load cell so that the force variation could be recorded.

By recording the input displacement and the output force, the calibration function F/X and the relative phase angle could be plotted as a function of frequency. Also by recording pressure variations in the tire, the calibration function F/P and the relative phase angle could be obtained as a function of frequency.

Several objections may be raised in connection with steady state sinusoidal calibration of a vehicle.

First, to build the equipment needed for applying this kind of calibration to a truck will be very expensive. Also the time needed for performing a calibration of this nature will be long when the response is investigated for a wide range of frequencies.

Wilson[9] pointed out some troublesome problems associated with the steady state sinusoidal calibration of a vehicle. If a constant amplitude of the input displacement was selected in order to obtain a relative large force amplitude for the low frequencies, the tire was likely to loose contact with the force measuring platform when the frequency of the input was increased. The result was then that the input displacement had to be decreased because the

force generated was proportional to the square of the frequency.

Since calibration tests show that the F/X and the F/P calibration functions are amplitude dependent, some nonlinear elements are present in the vehicle system. The causes of the detected nonlinear behavior may be due to Coulomb friction in the syspension springs, nonlinear effects of the shock absorbers and nonlinearities caused by the contact area of the tire. To minimize the effect of the nonlinear elements, the calibration conditions should be as close as possible to the operating conditions of a vehicle on a highway. Wilson[9] raised several questions about how the amplitude should be selected in a steady state calibration as indicated by the following quotation:

> "Should the displacement amplitude be selected so that the forces are always in a range corresponding to the dynamic force range on an "average" highway? Should the displacement amplitude be selected to produce force amplitudes which approximate the force amplitudes that are generated within each corresponding frequency band for "average" highway testing conditions? Should the displacement amplitudes be selected on the basis of the displacement amplitudes obtained from harmonic

> analysis of a "typical" highway? For each question above, a different displacement amplitude versus frequency relationship could be selected. To minimize the effect of the nonlinearity, calibration conditions must be as near the operating conditions as possible. But how can a single amplitude at a single frequency be made to correspond to a multitude of superimposed frequencies which exist under actual highway test conditions? It is apparent that the answers to these questions are not obvious. No one criterion for selecting amplitudes has been found that will satisfy all the requirements for a "perfect" calibration test."

The second calibration method that can be used to determine the frequency response of a system is to excite the system with a transient or aperiodic disturbance and then analyse the transient input and response of the system variables.

Hamilton[5] built a transient test device for use with passenger vehicles. The device was a pivoted beam with a load cell at the center. When a test was performed the vehicle was driven up on the beam with one wheel resting on the load cell. The beam was raised and then suddenly dropped. This produced a step input to the wheel.

If the raising and lowering was done in one continuous motion, it gave a pulse input to the wheel. Later McLemore[6] built a similar device to be used for truck calibration. This device can be used for producing a step input to a wheel of a truck.

An illustration of a typical step input and the resulting transient records of the dynamic force and the tire pressure are shown in Figure 4.

The Fourier transform analysis which transforms the aperiodic input and the transient response from the time domain to the frequency domain is used to determine the frequency characteristics of the vehicle or the calibration functions which are of interest (F/X and F/P).

Hamming[10] gives the following relationship for the Fourier transform of a function from the time domain to the frequency domain:

$$Z(f) = \int_{-\infty}^{+\infty} z(t) e^{-j2\pi ft} dt$$

where

$Z(f)$ = Fourier transform of the function $z(t)$

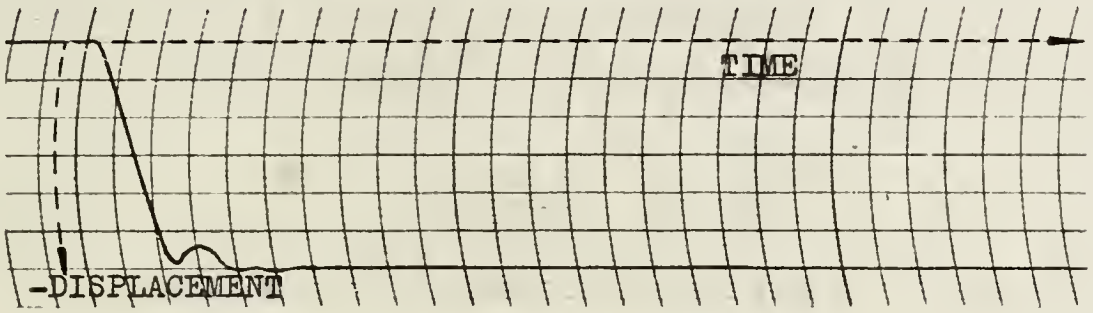
$z(t)$ = time domain function

f = frequency

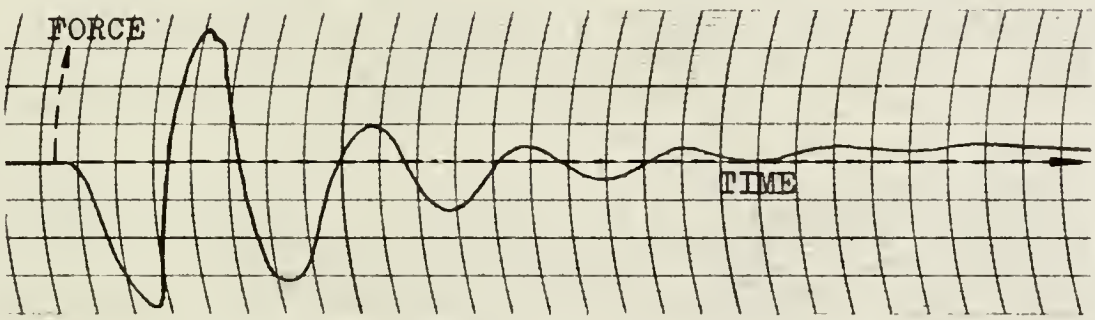
t = time

j = $\sqrt{-1}$

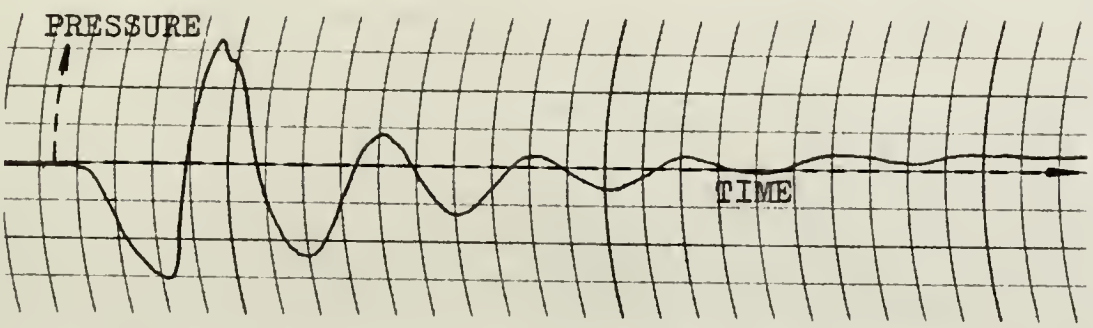
Hamming points out that $Z(f)$ obtained in this manner is a



RECORD OF TYPICAL STEP DISPLACEMENT INPUT



RECORD OF THE DYNAMIC FORCE TRANSIENT RESPONSE



RECORD OF THE TIRE PRESSURE TRANSIENT RESPONSE

FIGURE 4. ILLUSTRATION OF A STEP DISPLACEMENT INPUT AND THE RESULTING DYNAMIC FORCE AND TIRE PRESSURE RESPONSES

density function which describes the amount of the frequency that is present in the function z(t). The Z(f) function is a complex density function, and thus both the magnitude and the phase angle are obtained in the analysis. Using

$$e^{-j2\pi ft} = \cos 2\pi ft - j \sin 2\pi ft$$

then

$$Z(f) = \int_{-\infty}^{+\infty} z(t) \cos(2\pi ft)dt - j\int_{-\infty}^{+\infty} z(t) \cos(2\pi ft)dt$$

$$= a(f) - jb(f)$$

the phase angle $\theta_Z(f)$ is thus

$$\theta_Z(f) = \arctan\left(-\frac{b(f)}{a(f)}\right)$$

The absolute value of the density function, $|Z(f)|$, can then be expressed as

$$|Z(f)| = +\sqrt{a^2 + b^2}$$

therefore

$$Z(f) = |Z(f)|\, e^{[j\theta_Z(f)]}$$

Before proceeding further, an explanation will be given of the terms that are to be used in this thesis. The absolute value of the density function of a displacement

transient excitation will be called the input displacement density (inches/cycles/second). The absolute value of the density functions of the force and pressure records will be referred to as force amplitude density (lbs/cycles/second), and pressure amplitude density (psi/cycles/second). The term, amplitude density, will be used to describe the absolute value of the density function of any time history for a specific frequency.

Realizing that the Fourier transform defined in the above manner gives an amplitude density ordinate for any specific frequency in the frequency domain, the term frequency amplitude will be used to describe the area under the frequency domain curve in any frequency range. Thus, if it is desired to point out that the force amplitude density is high for all frequencies in a certain frequency range, this might be said as: The frequency amplitude in this frequency range is high. The unit of the frequency amplitude in this case will be lbs. If the calibration function F/X is required as a function of frequency, then:

$$\frac{F}{X}(f) = \frac{|F(f)|}{|X(f)|}$$

The relative phase angle between force and displacement, $\theta_R(f)$ is

$$\theta_R(f) = \theta_F(f) - \theta_X(f)$$

where

$\theta_F(f)$ = phase angle of force

$\theta_X(f)$ = phase angle of displacement.

In the same manner the calibration function F/P and the relative phase angle between force and pressure can be determined.

When a transient calibration is performed using a step input, the displacement input and the resulting force response do not reach a zero final value. A function z(t) is not Fourier transformable if

$$\int_{-\infty}^{+\infty} z(t)\, dt = \infty$$

Therefore one should conclude that the x(t) and f(t) functions are not Fourier transformable. However, it can be shown[2] that if the function z(t) reaches a constant value z(T) at time equal to T, the Fourier transform of the function will be:

$$Z(f) = \int_{o}^{T} z(t)e^{-j2\pi ft}dt - \frac{jz(T)e^{-j2\pi fT}}{2\pi f}$$

The frequency content of a displacement step input is shown in Figure 5.

It should be pointed out that for the transient calibration method the equipment needed is simple to build

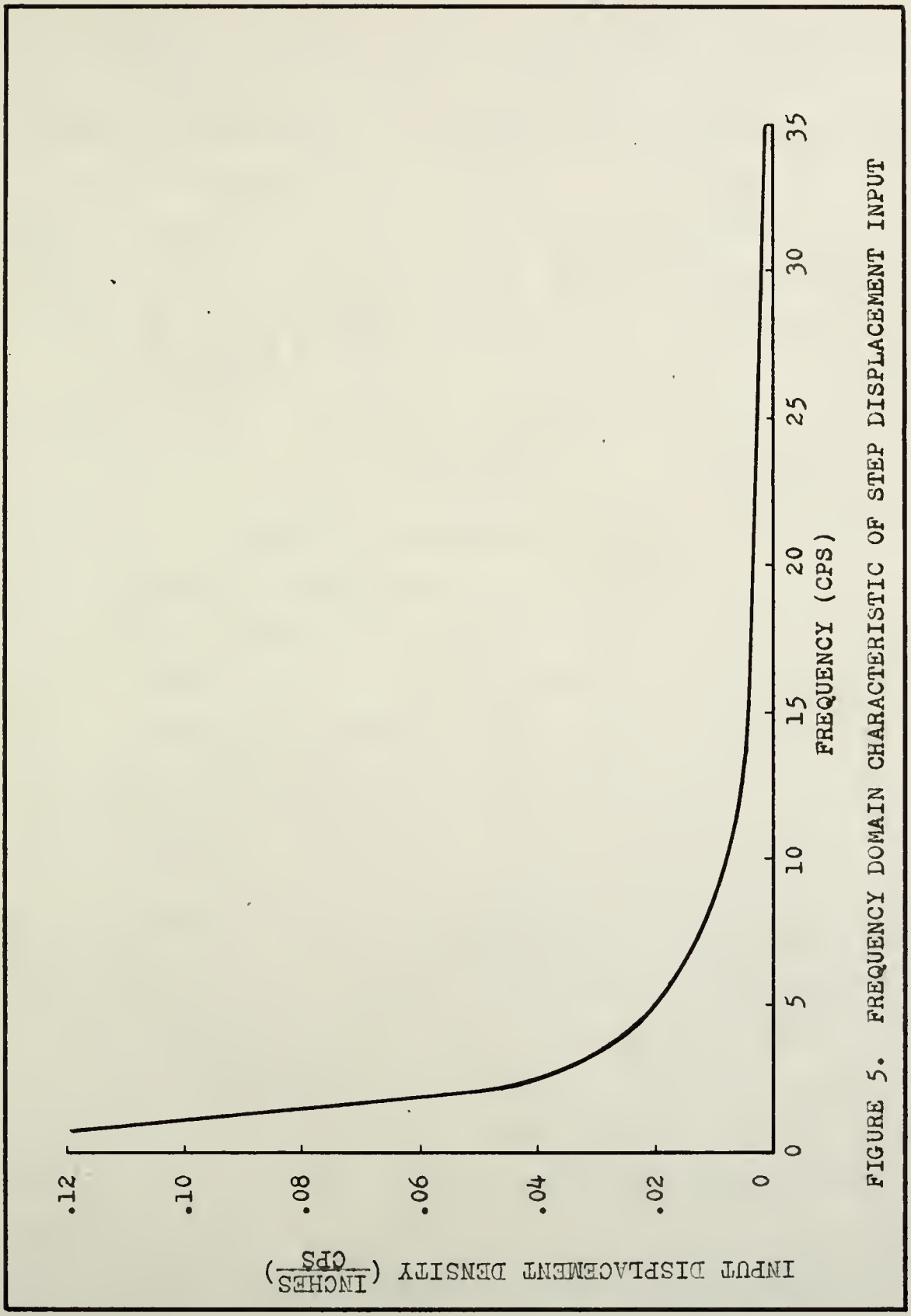


FIGURE 5. FREQUENCY DOMAIN CHARACTERISTIC OF STEP DISPLACEMENT INPUT

and operate. In theory a step or pulse input would excite all frequencies of the system. Thus only one calibration test should be necessary to determine the frequency characteristic of the calibration functions. With a digital computer available to perform the harmonic analysis, the calibration time is reduced to the actual test time plus the time needed to read and punch the calibration data.

Hamilton[5] maintains that the transient test calibration yields more correct results than the steady state test since the transient displacement input will have a continous frequency spectrum, and therefore more closely will simulate the characteristic of the motion input that the vehicle will experience on a highway. Since the transient calibration procedure has advantages over the steady state calibration procedure, an investigation of this technique to determine the frequency characteristics of heavy vehicles is desirable.

The transient calibration method is not without problems, however. Figure 5 shows that the input displacement density of the step displacement decreases rapidly as the frequency increases. For frequencies higher than 10 cps the input displacement density is very low. The question might be raised of how well the step input is exciting the force and pressure response in the higher frequency range.

The same question might be raised for the pulse in-

put since the frequency domain description is very similar to that of a step input.

Will it be possible to change the shape of the step and pulse input in order to get more frequency amplitude in the higher frequency region? If this is the case, what must the characteristics of the pulse or the step input be? If this is not the case what kind of input(s) will be needed? Can these "improved" inputs be applied to a physical system like a vehicle?

An illustration of the F/P calibration function of a truck for different heights of the step input is shown in Figure 6. Comparison of the calibration functions shows that they are very similar in the lower frequency range (0-4 cps), but for higher frequencies the variation is quite large. Is this only a result of the low frequency amplitude of the step input in the higher frequency rangeor do the nonlinearities of the system have a greater effect in the higher frequency range? Answers to these questions are needed in order to develop accurate calibration procedures based on the use of transient excitation.

The actual input to a wheel of a vehicle from a highway is of a random nature. Therefore, a random calibration procedure should give a relatively realistic frequency domain description of the response of a vehicle. This method can be used if power spectral density functions of the input displacement and the resulting dynamic force can

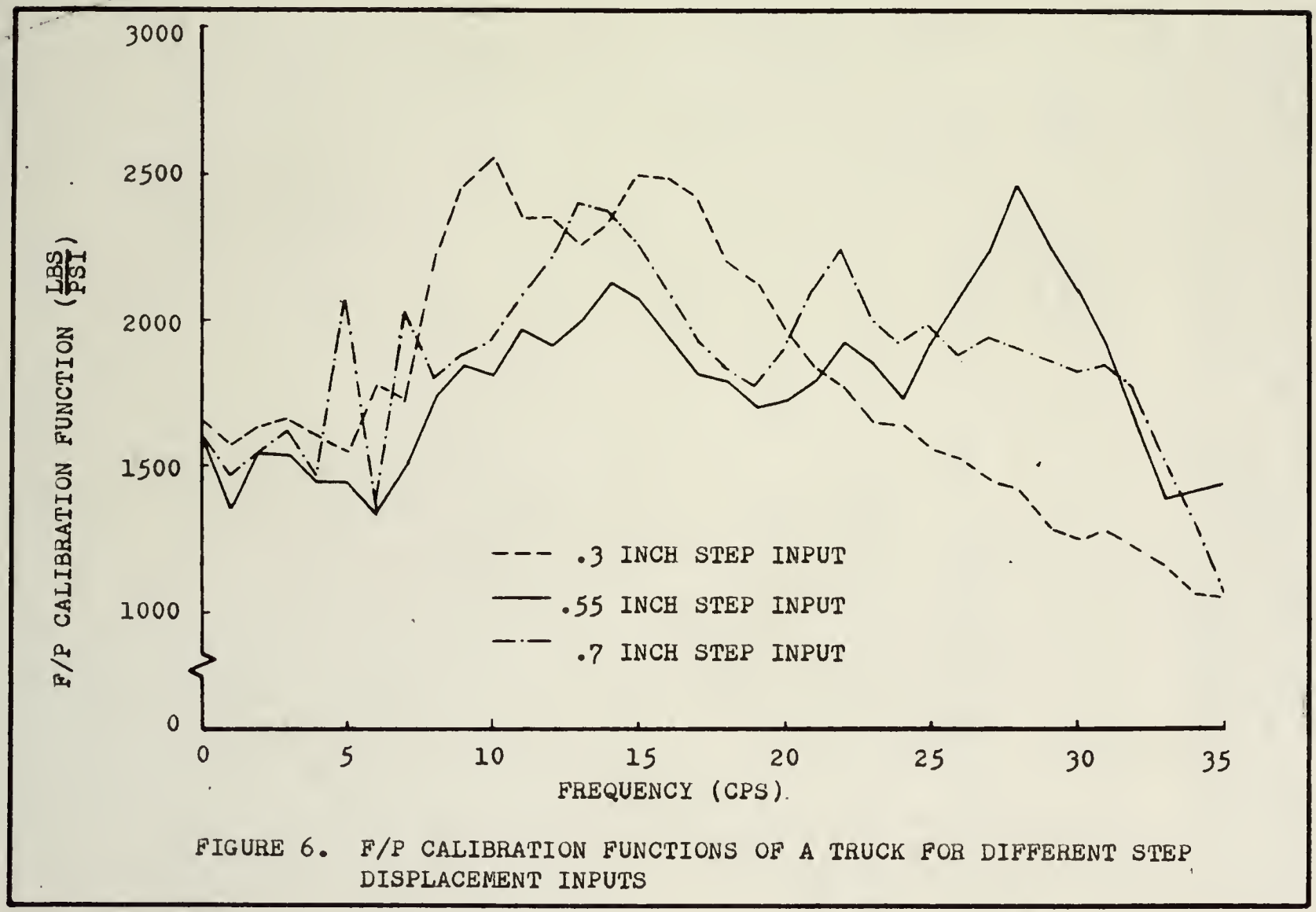


FIGURE 6. F/P CALIBRATION FUNCTIONS OF A TRUCK FOR DIFFERENT STEP DISPLACEMENT INPUTS

be obtained. If $P_F(f)$ is the power spectral density function of the tire force, and $P_X(f)$ is a power spectral density function obtained by using the highway elevation measurements, then

$$\frac{F}{X}(f) = \sqrt{\frac{P_F(f)}{P_X(f)}}$$

where F/X is the frequency domain calibration relationship that is desired.

Although $P_F(f)$ can be readily obtained, the technique for obtaining $P_X(f)$ requires further refinement before this procedure can be used to obtain calibration relationships of sufficient accuracy.

CHAPTER 3

INVESTIGATION OF TRANSIENT EXCITATIONS AVAILABLE FOR VEHICLE CALIBRATION

Two displacement inputs are at present available when a transient excitation is needed to determine the desired characteristics of a vehicle. These are:

1. A step input
2. A pulse input

When a dynamic calibration test of a vehicle is performed, two pronounced natural frequencies will be detected. The low natural frequency (1-2 cps) is due to vehicle body motion and is called the body motion frequency. The other natural frequency (15-17 cps) is commonly known as the wheelhop frequency. In the wheelhop mode the vehicle body is almost completely isolated from the disturbance of the wheel.

However, it was stated in the previous chapter that the frequency amplitude of the two available transient inputs in the higher frequency range is very small. Therefore, it is possible that there exist other natural frequencies of the vehicle or the pressure measuring system

which cannot be detected due to low input excitation.

Another problem which is connected with low frequency amplitude in the higher frequency range should also be pointed out. If the input excitation is low in a particular frequency range, then the relative frequency amplitude error due to reading of the oscillograph records is greater than in a frequency range where input excitation is larger. The reason is that for a fixed reading error of the time history, the reading error will have a greater effect on a small amplitude than on a large amplitude. Thus as the input displacement density decreases, it is clear that the confidence in the accuracy of the results at frequencies for which the input excitation is small will also decrease. This will affect the validity of the calibration ratios F/X or F/P. In a frequency range where the frequency amplitudes of F, X, and P are large, the calibration ratios can be determined with high accuracy. However, if frequency amplitudes of F, X or P are low, the validity of the calibration ratios may be questioned.

What is the minimum amplitude density at any frequency that can be accepted when a transient test is performed? An obvious answer to this question is not evident. However, the reading accuracy of the time histories and the sampling rate at which the data are read must be taken into account, if an attempt is made to give an answer. Dreifke[3] gives the following criteria, allowing for about

25% maximum error in the experimental frequency response:

$$|F_1(f)| > 8e$$

$$|F_2(f)| > 8e$$

$$f_{max} < \frac{1}{4\Delta t}$$

$$\Delta t \leq \frac{T_1}{10}$$

where

$|F_1(f)|$ = amplitude density of the input at frequency f

$|F_2(f)|$ = amplitude density of the response at frequency f

e = estimated ordinate reading error

f_{max} = maximum frequency for which the frequency content should be estimated (cps)

Δt = time interval (sec) for reading of data

T_1 = smallest characteristic time of interest in the system

From the above discussion it is clear that more accurate results will be obtained for the calibration ratios at any specific frequency if the amplitude density of the two transient functions at this frequency is large.

How can the "shapes" of the impulsive excitation

inputs be changed to obtain a more desirable frequency domain characteristic?

One way to attack this problem would be to obtain a known analytical function having a time domain history similar to the experimental input displacement. Then a frequency domain comparison could be made, and if the two frequency domain descriptions could be shown to be similar, the analytical function would represent the experimental function very well. By changing the "shape" of the analytical function and observing the corresponding change in the frequency domain, a desirable frequency domain description might be obtained. The experimental input can then be changed to correspond with the "improved" analytical function.

An analytical function having a "shape" similar to an experimental step function in the time domain is of the following form:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = \frac{a}{2}\left[\cos\left(\frac{2\pi}{T} t\right) - 1\right] , \quad 0 \leq t \leq \frac{T}{2}$$

$$x(t) = -a , \quad t > \frac{T}{2}$$

where

t = time variable (seconds)

a = drop height for the step input (inches)

$\frac{T}{2}$ = time for calibrator drop beam to drop the distance "a" (seconds)

A comparison between the analytical function and the experimental step input in the time domain is shown in Figure 7. The corresponding comparison in the frequency domain is given in Figure 8.

Since the comparison in the frequency domain shows excellent correlation, the analytical function must be said to represent the experimental step input very well.

The frequency amplitude density of the analytical function, when $f \neq \frac{1}{T}$, will be:

$$X(f) = \frac{jaT}{4\pi}\left\{\frac{1}{fT - (fT)^3}\left[\cos(\pi Tf) - j\sin(\pi Tf) + 1\right]\right\}$$

If $f = \frac{1}{T} = f_1$, the following equation should be used:

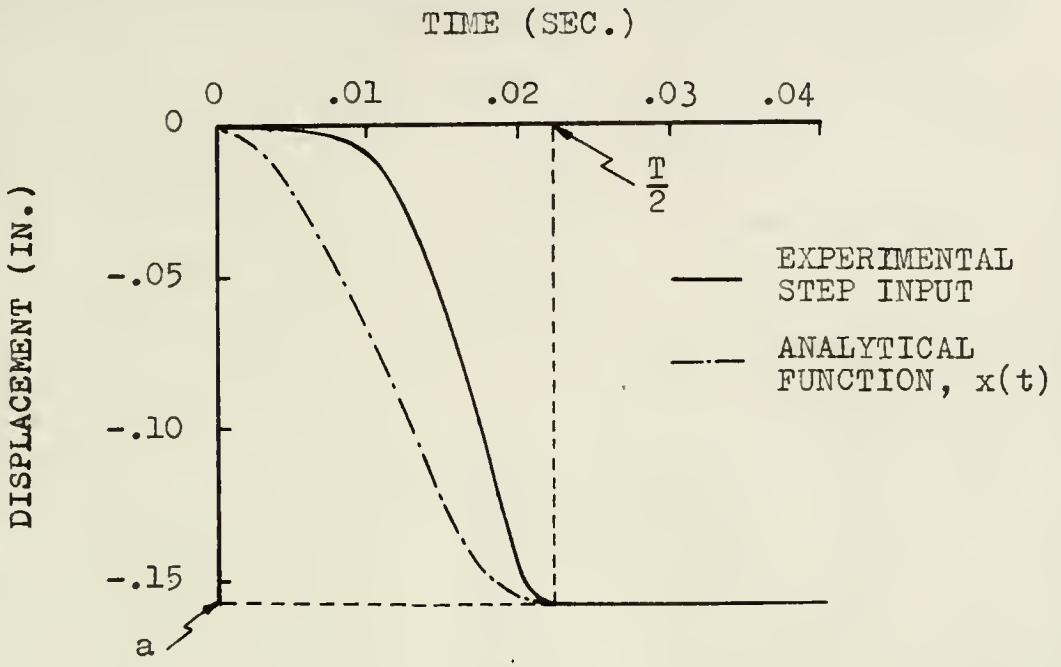
$$X(f_1) = \frac{aT}{8}$$

The equation for X(f) can also be written as:

$$\frac{X(f)}{aT} = \frac{j}{4\pi}\left\{\frac{1}{fT - (fT)^3}\left[\cos(\pi Tf) - j\sin(\pi Tf) + 1\right]\right\}$$

The units of X(f) and f are inches/cycles/second and cycles/second respectively. Therefore, $\frac{X(f)}{aT}$ has units of 1/cycles, or dimensionless, and fT has units of cycles or dimensionless.

The frequency characteristics of the analytical step function and the experimental step displacement can now be



THE ANALYTICAL FUNCTION IS GIVEN BY:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = \frac{a}{2}\left[\cos\left(\frac{2\pi}{T} t\right) - 1\right] , \quad 0 \leq t \leq \frac{T}{2}$$

$$x(t) = -a , \quad t > \frac{T}{2}$$

FIGURE 7. COMPARISON OF AN ANALYTICAL AND AN EXPERIMENTAL STEP DISPLACEMENT INPUT IN THE TIME DOMAIN

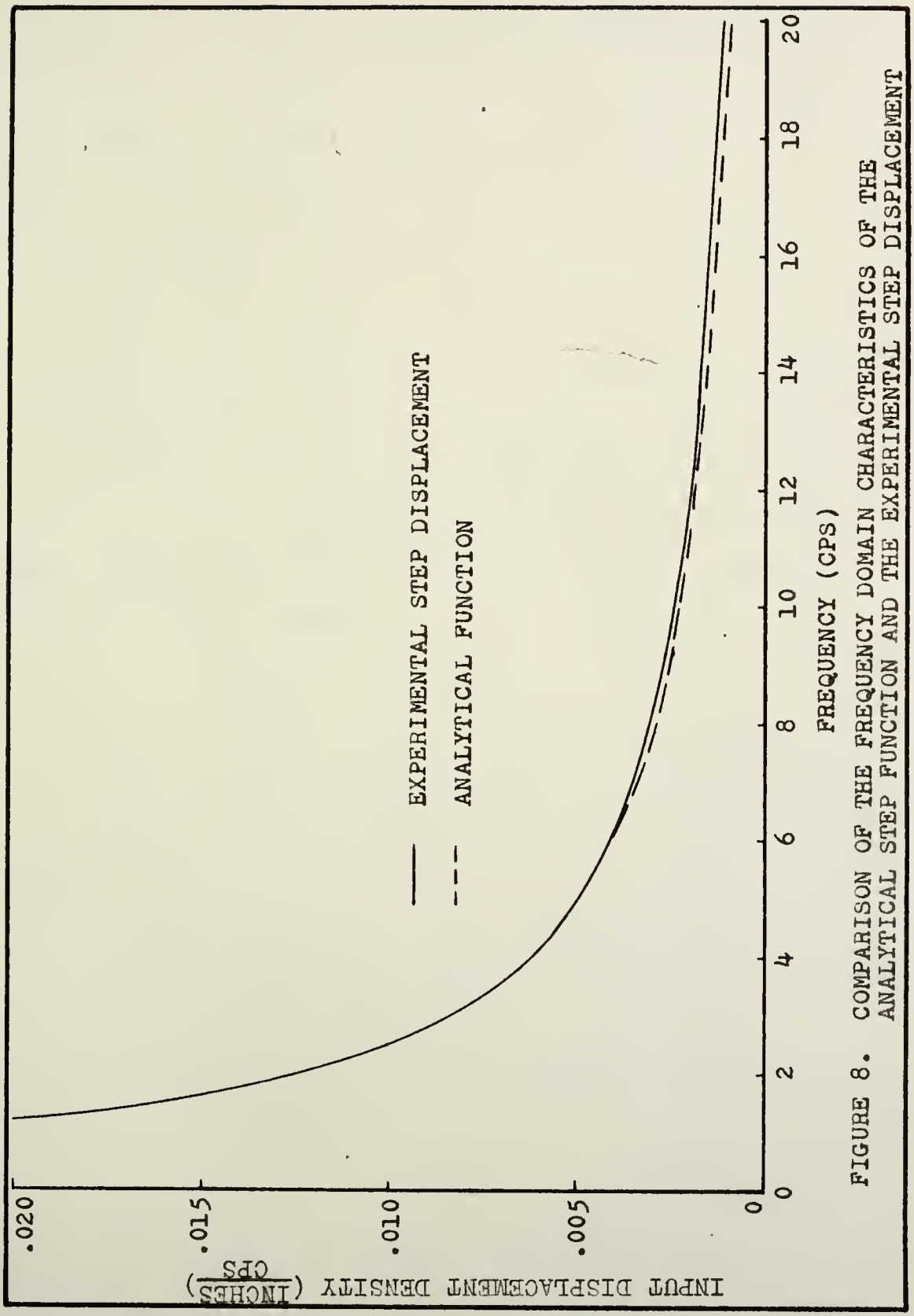


FIGURE 8. COMPARISON OF THE FREQUENCY DOMAIN CHARACTERISTICS OF THE ANALYTICAL STEP FUNCTION AND THE EXPERIMENTAL STEP DISPLACEMENT

plotted in dimensionless form $X(f)_d$ as a function of the dimensionless quantity f_d , where

$$X(f)_d = \frac{X(f)}{aT}$$

and

$$f_d = fT$$

An illustration of $X(f)_d$ of the analytical and the experimental step versus f_d is shown in Figure 9.

To find the frequency domain characteristic of any step displacement input with a similar shape as the original experimental input shown in Figure 7, the following manipulations must be made:

$$f = \frac{f_d}{T_n}$$

$$X(f) = X(f)_d \, a_n \, T_n$$

where

$\frac{T_n}{2}$ = the time for drop beam to drop the distance "a_n" (seconds)

a_n = the drop height of the step displacement (inches)

By performing an analysis of this type, the frequency characteristics of four different step displacements were determined. The four different step displace-

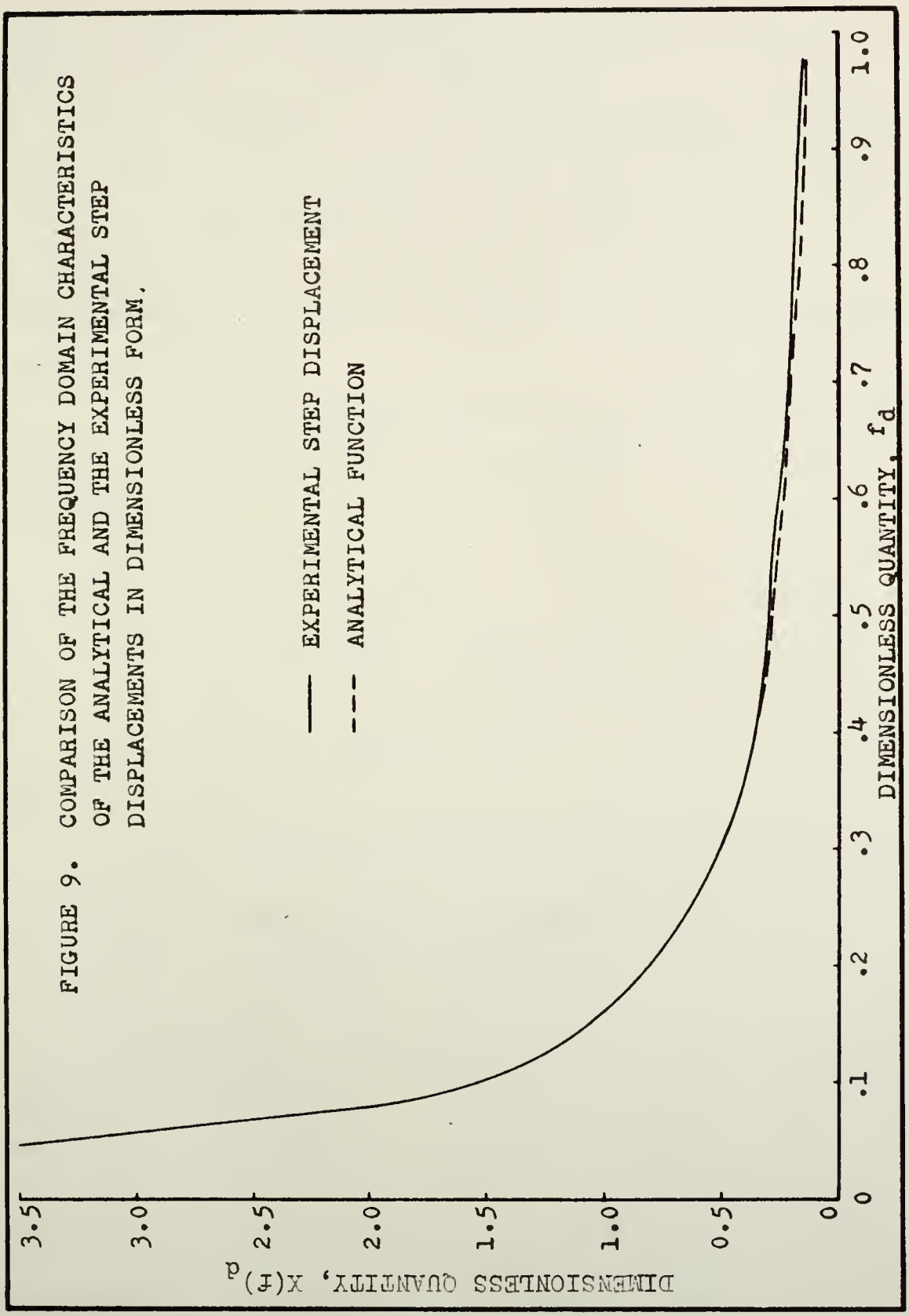


FIGURE 9. COMPARISON OF THE FREQUENCY DOMAIN CHARACTERISTICS OF THE ANALYTICAL AND THE EXPERIMENTAL STEP DISPLACEMENTS IN DIMENSIONLESS FORM.

ments in the time domain and in the frequency domain are shown in Figures 10 and 11 respectively.

These two figures show that the frequency amplitude increases with increasing drop height and that the drop time is relatively unimportant. This means that in order to increase the input displacement density at all frequencies using an experimental step input, the drop height should be increased.

An analytical function that can be compared with the experimental pulse input in the time domain can be of the form:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = b \sin \frac{2\pi}{T_1} t , \quad 0 \leq t \leq \frac{T_1}{2}$$

$$x(t) = 0 , \quad t > \frac{T_1}{2}$$

where

b = height of experimental pulse (inches)

$\frac{T_1}{2}$ = time duration of the experimental pulse (seconds)

A comparison of the analytical function and the experimental pulse input is given in Figure 12. The corresponding comparison in the frequency domain is shown in Figure 13. Since the comparison of the two pulse displacements shows good correlation, the analytical function must be said to give a good representation of the

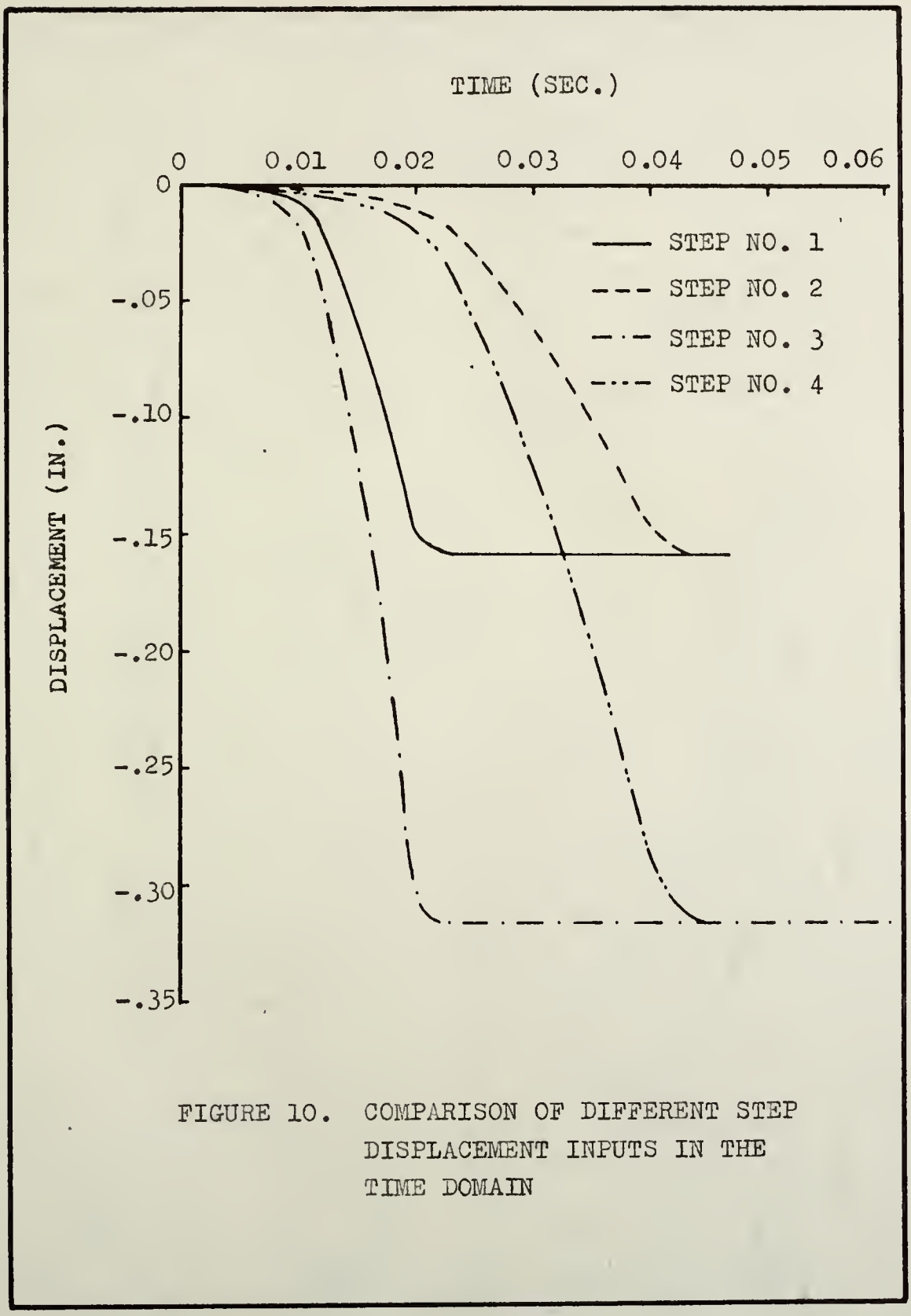


FIGURE 10. COMPARISON OF DIFFERENT STEP DISPLACEMENT INPUTS IN THE TIME DOMAIN

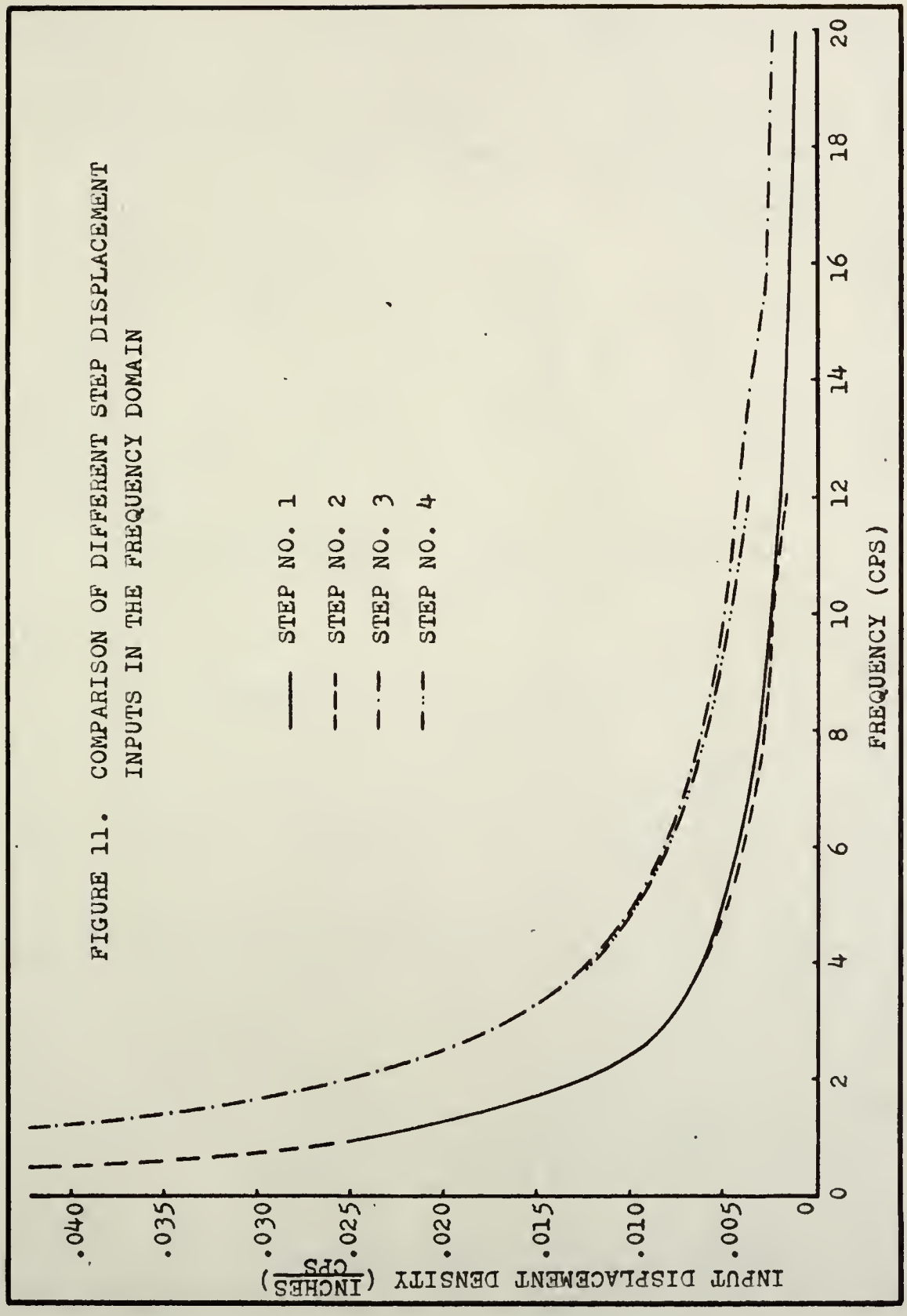
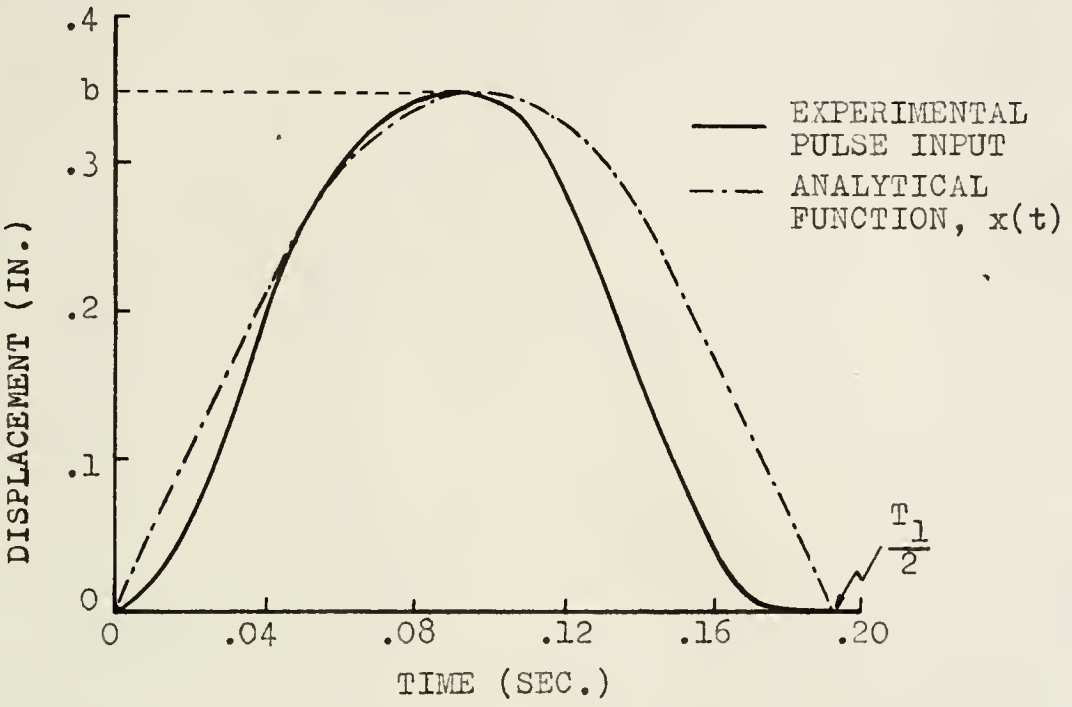


FIGURE 11. COMPARISON OF DIFFERENT STEP DISPLACEMENT INPUTS IN THE FREQUENCY DOMAIN



THE ANALYTICAL FUNCTION IS GIVEN BY:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = b \sin \frac{2\pi}{T_1} t , \quad 0 \leq t \leq \frac{T_1}{2}$$

$$x(t) = 0 , \quad t > \frac{T_1}{2}$$

FIGURE 12. COMPARISON OF AN ANALYTICAL AND AN EXPERIMENTAL PULSE DISPLACEMENT INPUT IN THE TIME DOMAIN

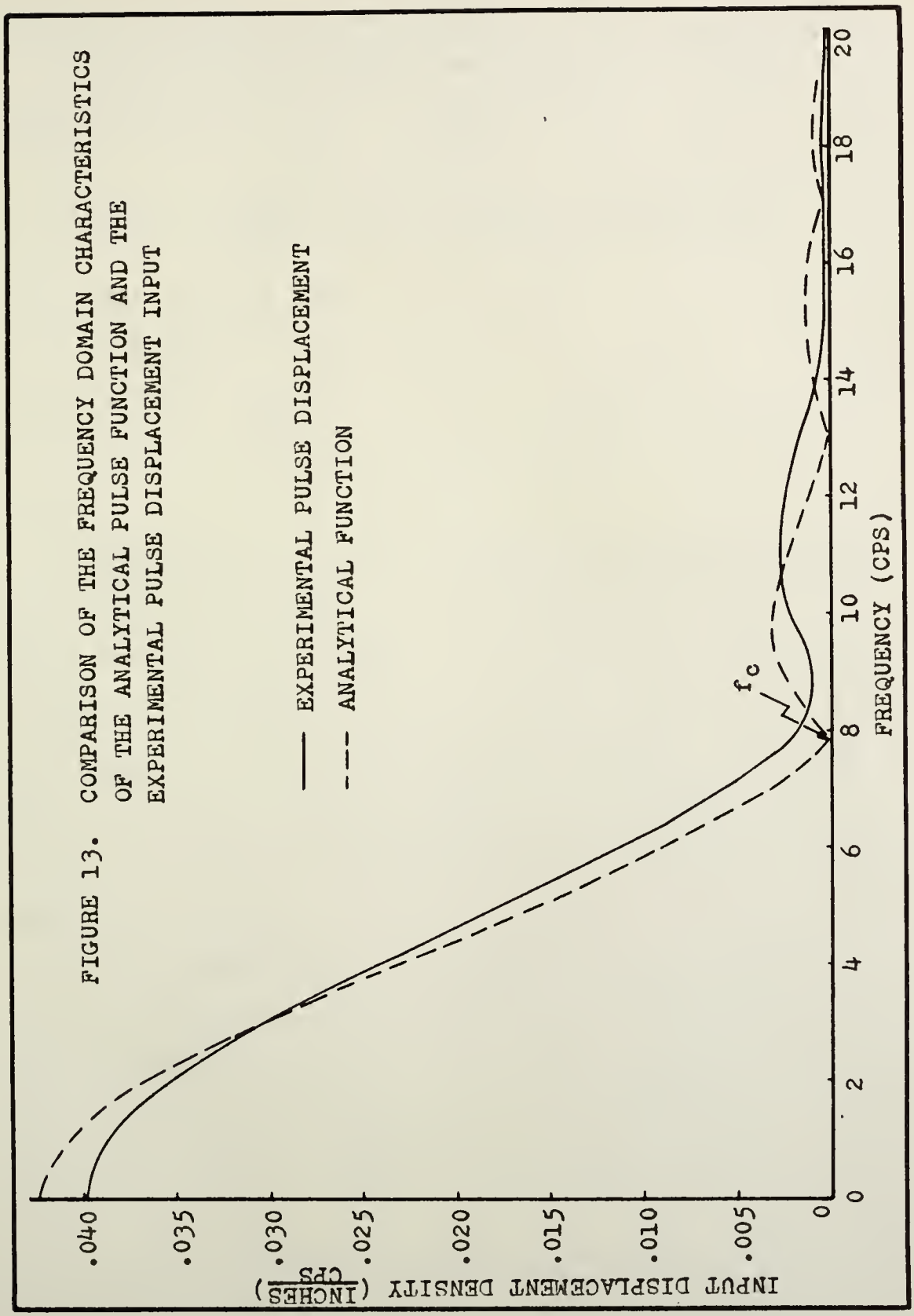

COMPARISON OF THE FREQUENCY DOMAIN CHARACTERISTICS
OF THE ANALYTICAL PULSE FUNCTION AND THE
EXPERIMENTAL PULSE DISPLACEMENT INPUT
EXPERIMENTAL PULSE DISPLACEMENT
ANALYTICAL FUNCTION
INPUT DISPLACEMENT DENSITY (INCHES / CPS)
.040
.035
.030
.025
.020
.015
.010
.005
0
2
4
6
8
10
12
14
16
18
20
FREQUENCY (CPS)
f_c

FIGURE 13.

experimental pulse displacement.

The frequency domain description of the above analytical function when $f \neq \frac{1}{T_1}$ will be:

$$X(f) = \frac{aT_1}{2\pi}\left[\frac{1 + \cos(\pi T_1 f) - j \sin(\pi T_1 f)}{1 - (T_1 f)^2}\right]$$

when $f = \frac{1}{T_1} = f_1$:

$$X(f_1) = \frac{aT_1}{4j}$$

Figure 13 shows that the input displacement density of the analytical pulse is greater than zero at zero frequency. However, the input displacement density decreases for increasing frequency, and will reach zero for $f = f_c$. For f greater than f_c the frequency amplitude of the analytical pulse input will be small. The most important frequency region of the analytical pulse function is thus between zero and f_c.

To determine f_c and the manner in which this frequency is influenced by the parameters of the analytical pulse function, a study of X(f) for the analytical pulse input must be made. Remembering that the equation of X(f) is not valid for $f = \frac{1}{T_1}$, it can be shown that

$$X(f) = 0$$

when

$$\pi T_1 f = 3\pi \ , \ 5\pi \ , \ 7\pi \ \ldots\ldots \text{ etc.}$$

or

$$f = \frac{3}{T_1} \ , \ \frac{5}{T_1} \ , \ \frac{7}{T_1} \ \ldots\ldots \text{ etc.}$$

Then

$$f_c = \frac{3}{T_1}$$

The "usable" frequency content of a pulse can thus be increased to cover a wider frequency range if T_1 is decreased. This means that a shorter pulse gives greater excitation over a wider range of frequencies than does a long pulse. However, it must be noted that X(f) is proportional to the amplitude of the pulse and to the time constant T_1. Thus as T_1 is decreasing one must increase the amplitude in order to get an adequate frequency amplitude over a wide span of frequencies.

To show how the frequency description of a pulse input changes, an analysis similar to the one performed with the step input displacement can be performed. The "dimensionless frequency content" $X(f)_d$ can be plotted against a "dimensionless quantity" f_d. (Figure 14) The frequency content of pulses having other amplitudes and time constants than the original pulse displacement can then be found by the same analysis as explained for the

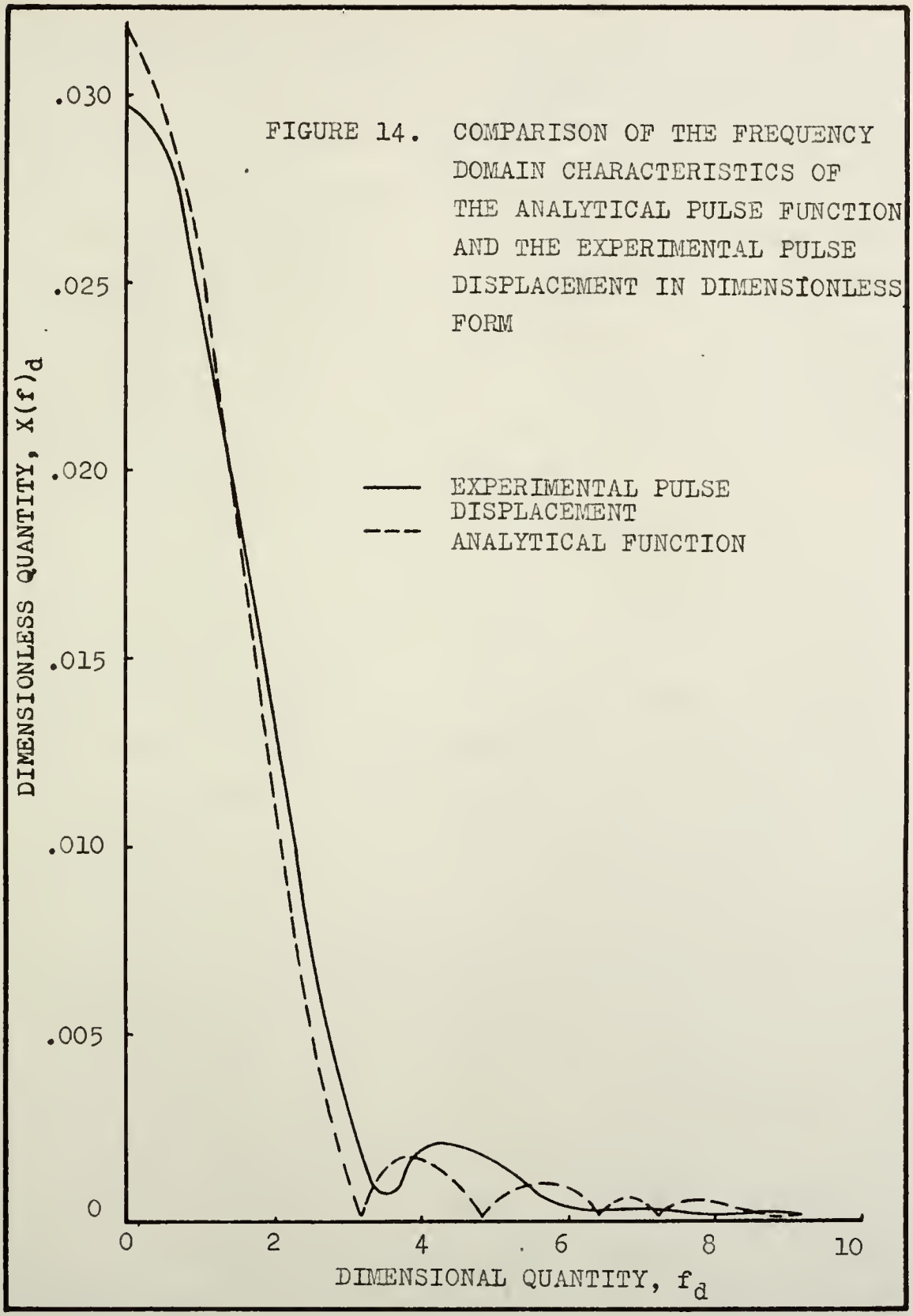


FIGURE 14. COMPARISON OF THE FREQUENCY DOMAIN CHARACTERISTICS OF THE ANALYTICAL PULSE FUNCTION AND THE EXPERIMENTAL PULSE DISPLACEMENT IN DIMENSIONLESS FORM

step displacement inputs.

The time histories of four different pulse displacements are shown in Figure 15 , and the corresponding frequency domain descriptions are given in Figure 16 . The two figures show that in order to get a more desirable frequency content, the pulse amplitude must be increased and at the same time, the time duration of the pulse must be decreased. This result is not surprising since the amplitude density of a unit impulse can be shown to be equal to unity for all frequencies.

If it is desired to test the frequency response for a selected band of frequencies, a multiple pulse might be used as the impulsive or transient excitation input. An analysis was made to calculate the frequency domain description of the multiple pulse shown in Figure 17.

The time domain description of the multiple pulse was:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = c \sin \frac{2\pi}{T} t , \quad 0 \leq t \leq n \frac{T}{2}$$

$$x(t) = 0 , \quad t > n \frac{T}{2}$$

where

n = number of half sine wave pulses

T = time for a complete sine wave (seconds)

c = amplitude of the pulses (inches)

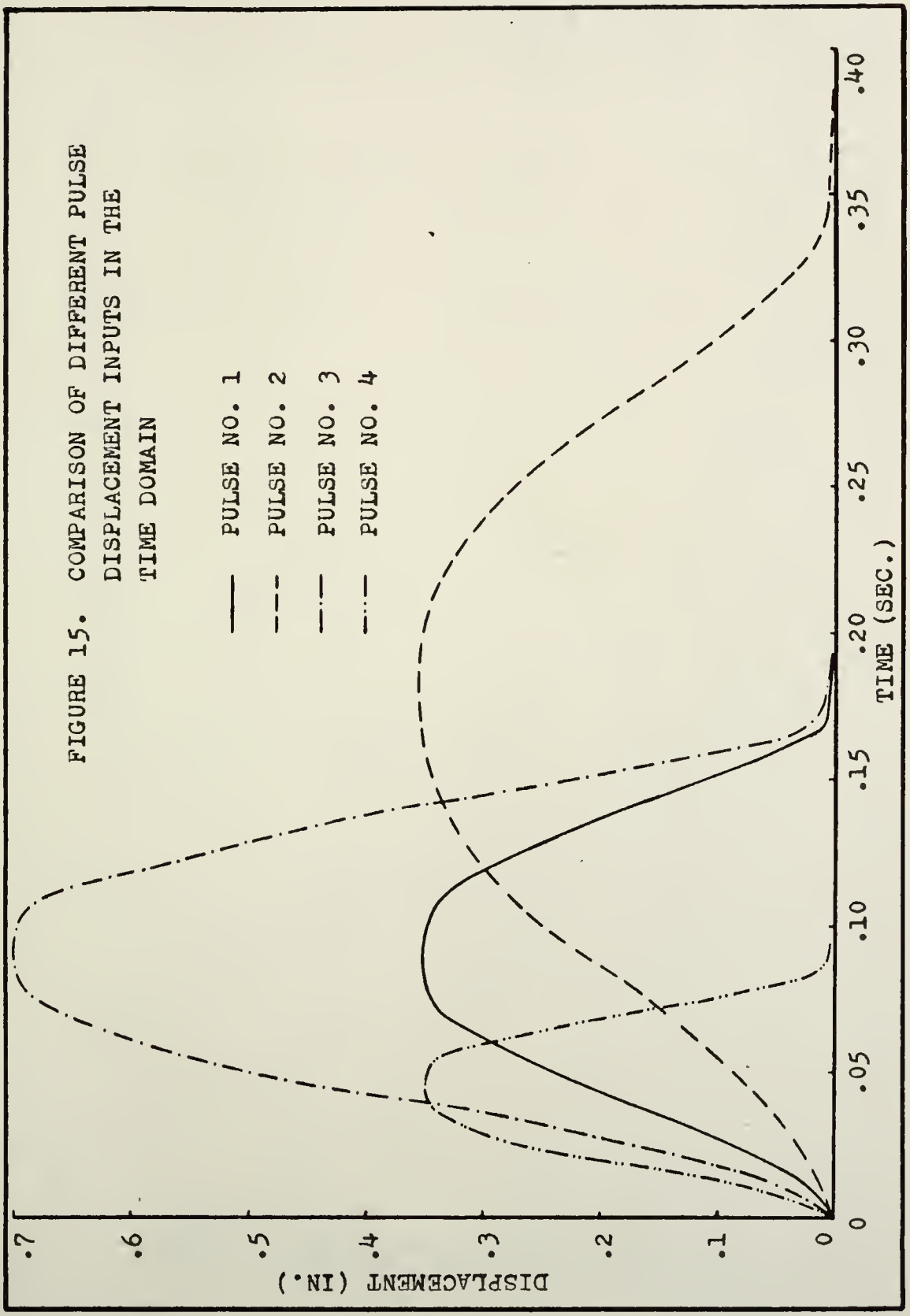


FIGURE 15. COMPARISON OF DIFFERENT PULSE DISPLACEMENT INPUTS IN THE TIME DOMAIN

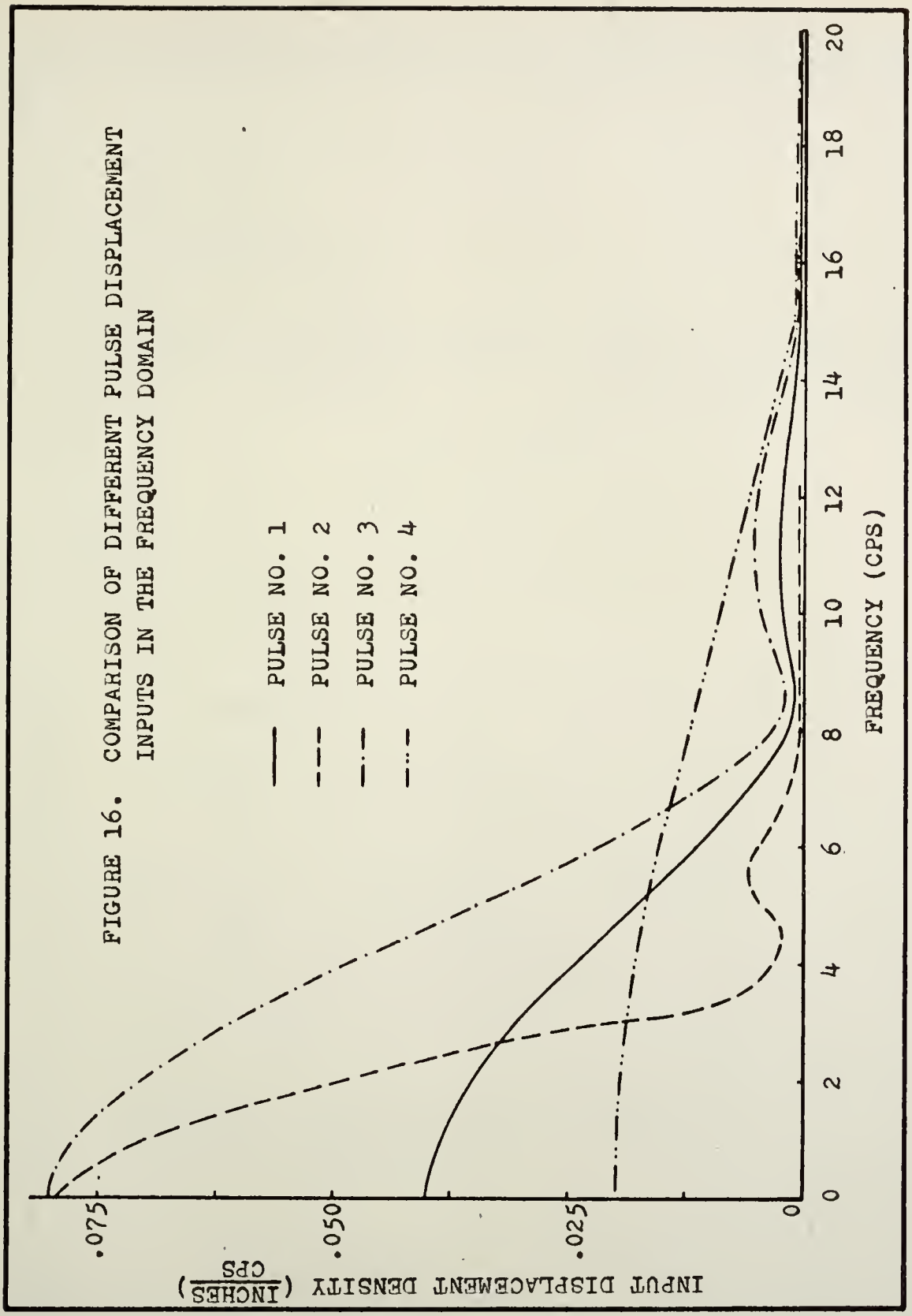
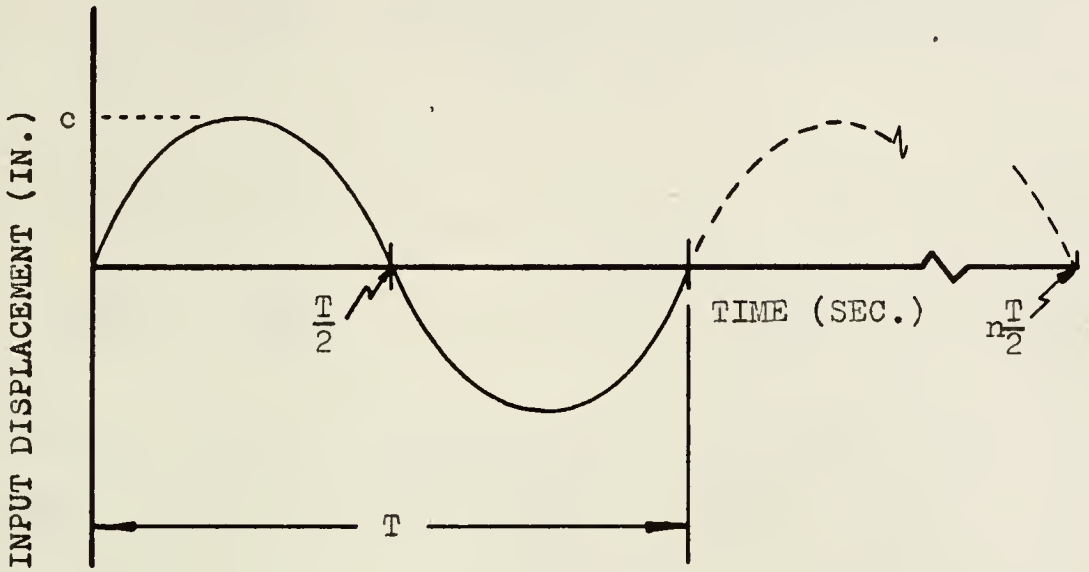


FIGURE 16. COMPARISON OF DIFFERENT PULSE DISPLACEMENT INPUTS IN THE FREQUENCY DOMAIN



THE MULTIPLE PULSE IS GIVEN BY:

$$x(t) = 0 , \quad t < 0$$

$$x(t) = c \sin \frac{2\pi}{T} t , \quad 0 \leq t \leq n\frac{T}{2}$$

$$x(t) = 0 , \quad t > n\frac{T}{2}$$

FIGURE 17. ILLUSTRATION OF A MULTIPLE PULSE INPUT

The frequency description of a multiple pulse when $f \neq \frac{1}{T}$ will be,

$$X(f) = \frac{cT}{2\pi} \left\{ \frac{\cos(n\pi) \left[\cos(n\pi Tf) - j \sin(n\pi Tf)\right] - 1}{(fT)^2 - 1} \right\}$$

when $f = \frac{1}{T} = f_1$

$$X(f_1) = \frac{cn}{j4f_1}$$

The frequency characteristic of a multiple pulse will always have a peak for $f \approx f_1$. Looking at the expression for $X(f_1)$ it is observed that the maximum amplitude density is proportional to the amplitude of the multiple pulse and to the number of pulses. On the other hand $X(f_1)$ is inversely proportional to f_1 which is the frequency of the multiple pulse. The amplitude c must be limited if an input of this type is to be applied to the wheel of a vehicle. This means that to produce a significant frequency content in the higher frequency region the number of pulses must be fairly large.

CHAPTER 4

EXPERIMENTAL RESULTS OBTAINED BY USING DIFFERENT TRANSIENT EXCITATIONS

It was stated in Chapter 3 that the "shape" of the step displacement could be changed to increase the input displacement density for any specific frequency by increasing the amplitude of the step input.

Figure 18 shows how the frequency domain characteristics of step inputs were changed by increasing the drop height in the time domain. Three step inputs with heights equal to 0.17 inch, 0.67 inch and 1.15 inch were used. Figure 18 clearly shows that the step displacement having the smallest height has the lowest input displacement density for all frequencies. The step input with the highest drop height has a higher input displacement density for all frequencies up to 28 cycles per second than the 0.67 inch step displacement. It would be logical to expect that the 1.15 inch step displacement would also have a higher input displacement density than the 0.67 inch step displacement for frequencies greater than 28 cps. However, it was explained in Chapter 3 that a fairly large frequency amplitude error could be expected in a region where the

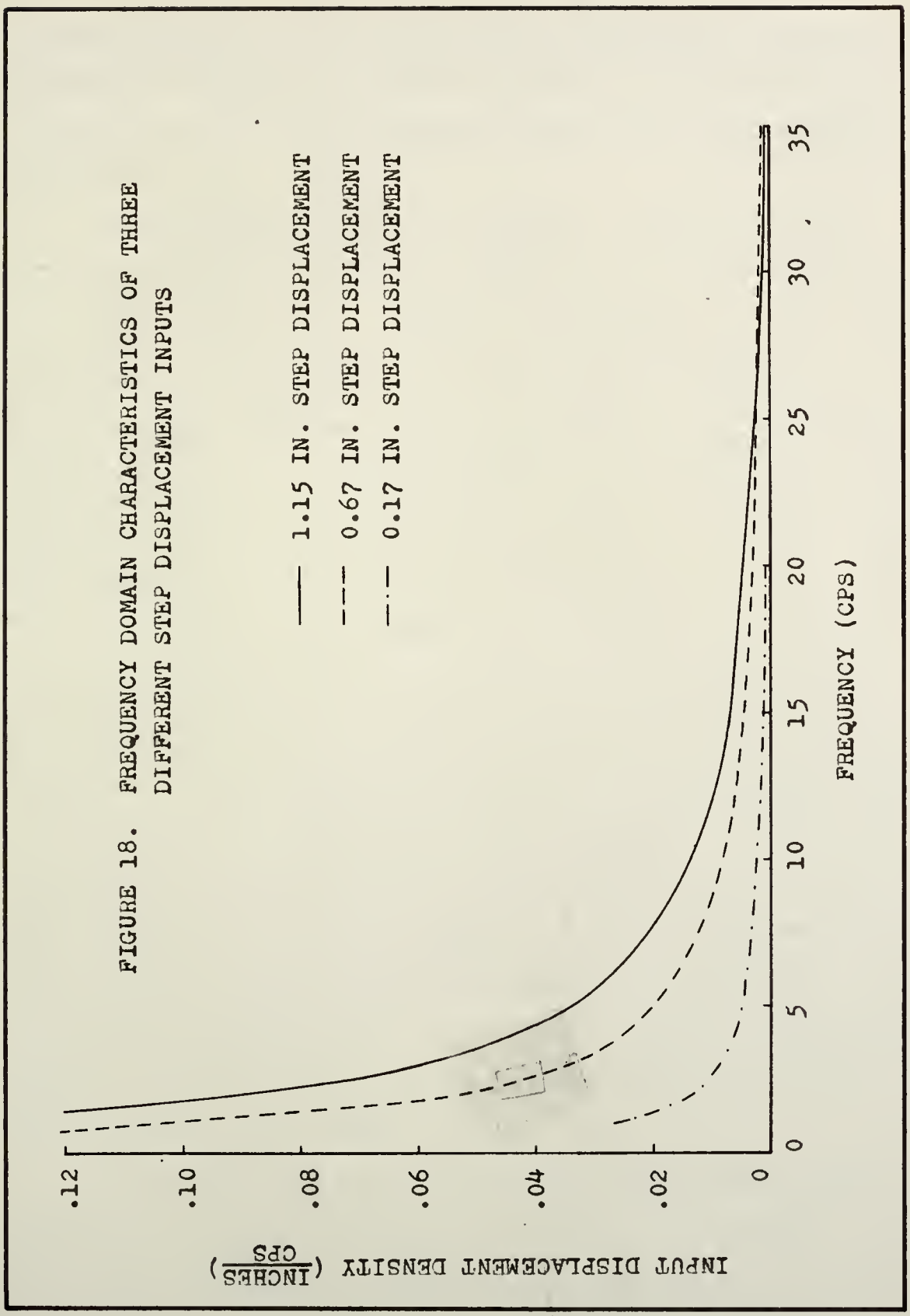


FIGURE 18. FREQUENCY DOMAIN CHARACTERISTICS OF THREE DIFFERENT STEP DISPLACEMENT INPUTS

frequency amplitude was low. This explains why the input displacement density for the 1.15 inch and 0.67 inch step displacement are almost equal for frequencies larger than 28 cps.

By increasing the drop height to higher and higher values the frequency amplitude should increase. In theory, this is of course true. However, limitations of the equipment used for recording the force variation limited the maximum height to approximately 1.15 inches. If the drop height was increased even further, the deceleration of the drop beam when it hit the base would result in a force too large to be recorded by the instrumentation.

To increase the "usable" frequency range of a pulse displacement input, the amplitude of the pulse should be increased and the time duration of the pulse should be decreased. Since the method used to give a pulse input to a passenger car depends upon manual dexterity, the time duration of the pulse cannot be very short. This means that the "usable" frequency band of a pulse displacement input obtained by the present calibration procedure is limited to low frequencies. A comparison in the frequency domain of a 1.04 inch pulse displacement and a 1.15 inch step displacement is shown in Figure 19.

The input displacement density and the force amplitude density at zero frequency are defined as follows:

$$X(0) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi 0}dt = \int_{0}^{+\infty} x(t)\,dt$$

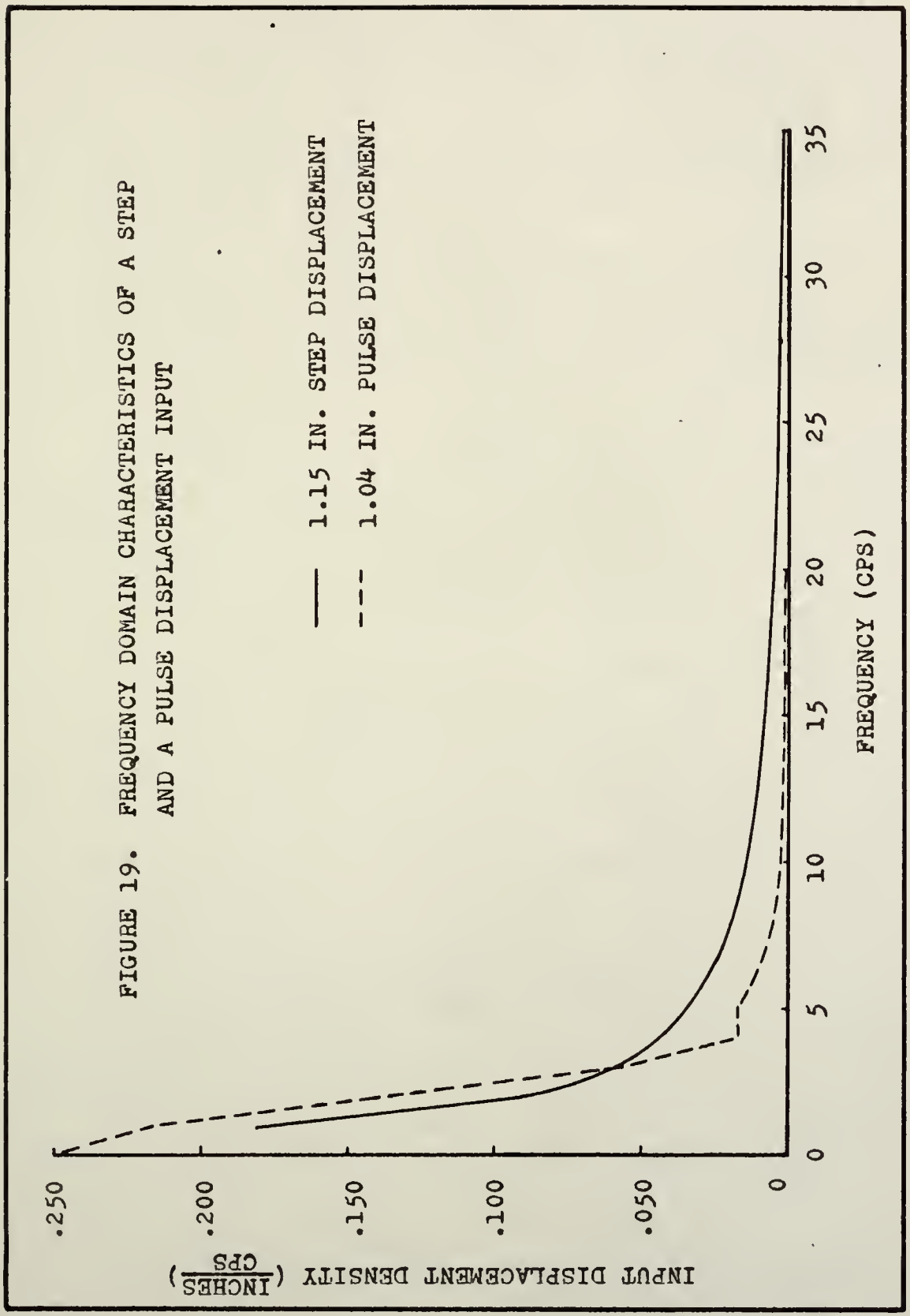


FIGURE 19. FREQUENCY DOMAIN CHARACTERISTICS OF A STEP AND A PULSE DISPLACEMENT INPUT

and

$$F(0) = \int_{-\infty}^{+\infty} f(t)e^{-j2\pi 0}dt = \int_{0}^{\infty} f(t)\,dt$$

Thus X(0) and F(0) will be equal to the area under the x(t) and f(t) curves. Since the step displacement and the resulting force do not reach a zero final value, the input displacement density and the corresponding force amplitude density at zero frequency should approach infinity.

This argument shows that it is impossible to evaluate the F/X calibration ratio at zero frequency when a step displacement input is used. This difficulty, however, does not exist when a pulse displacement input is used, since both the area under the displacement and force curves will have an exact finite value.

Figure 19 shows that the frequency amplitude of the pulse displacement in the 1-3 cps frequency range is larger than the corresponding frequency amplitude of the 1.15 inch step displacement.

When the transient calibration tests were performed, it was noted that the pulse displacement excited the body roll frequency of the vehicle more than did the step displacements. This indicates that a pulse displacement input should be used when the F/X calibration ratio is to be determined for the low frequency range (0-3 cps).

On the other hand, the 1.15 inch step displacement

gives a larger frequency amplitude than the pulse displacement when the frequency is greater than 3 cps. This indicates that the F/X calibration ratio obtained from a large step displacement should be used when the frequency is greater than 3 cps.

The frequency domain characteristics of pressure fluctuations resulting from the three step displacement inputs are shown in Figure 20. A comparison is given in Figure 21 of the pressure response resulting from the 1.15 inch step displacement and the 1.04 inch pulse displacement. Figure 21 shows that the pressure amplitude density for the pulse and the high step displacement inputs are about the same in the 0-3 cps frequency range. However, since the pulse displacement input excited the body roll frequency more than did the step inputs, it is believed that the pulse displacement should be used to determine the F/P calibration ratio from 0-3 cps. Another reason for using the pulse displacement input to evaluate the F/P calibration ratio in the 0-3 cps frequency range can be explained as follows. The force measuring system will measure the change in the static force due to a finite displacement of the wheel from its original position, and thus the recorded force will not go back to zero, but reach a final value when a step displacement input is used. However, the pressure record will show that the measured pressure fluctuations will reach a zero final value, caused by

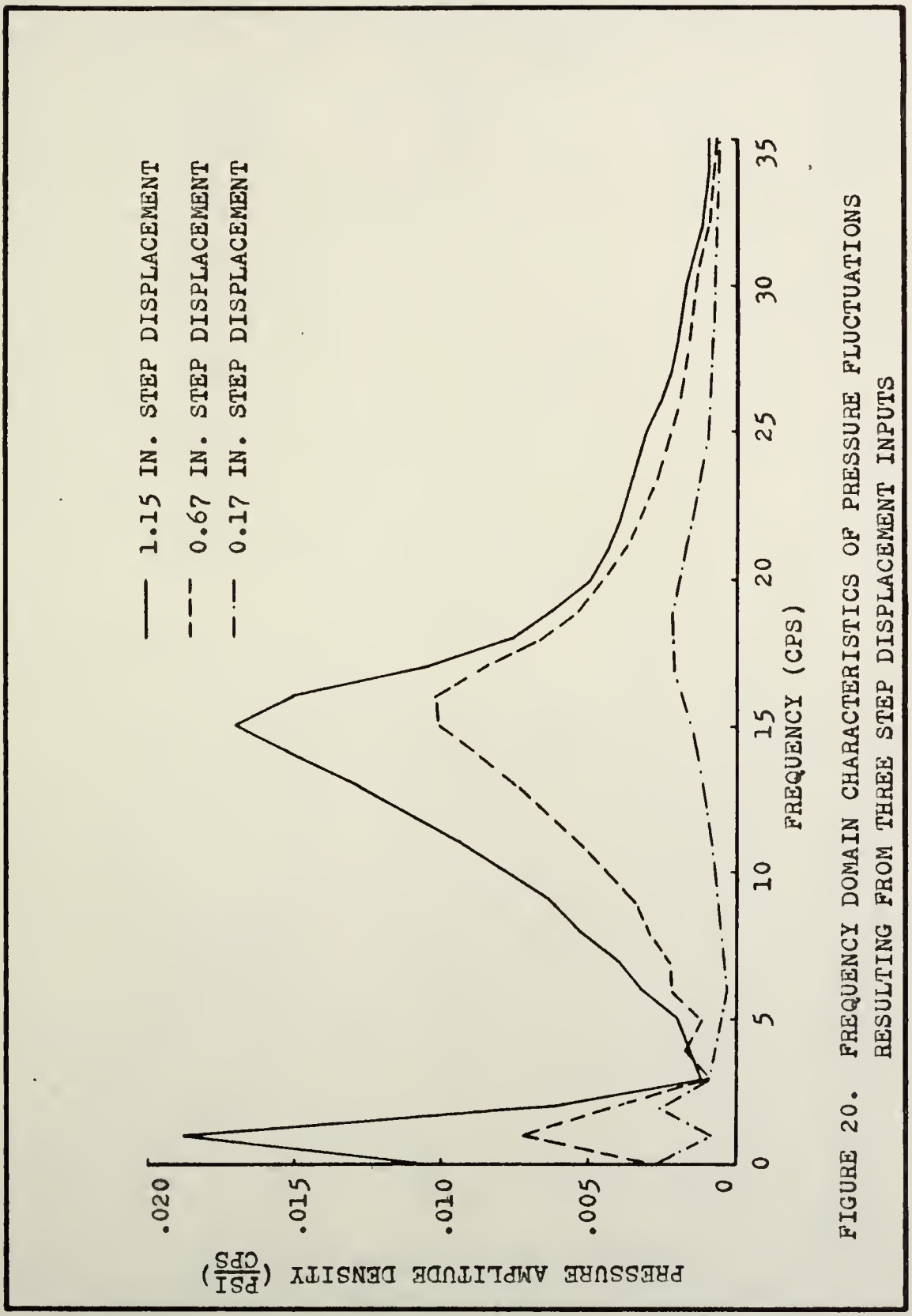


FIGURE 20. FREQUENCY DOMAIN CHARACTERISTICS OF PRESSURE FLUCTUATIONS RESULTING FROM THREE STEP DISPLACEMENT INPUTS

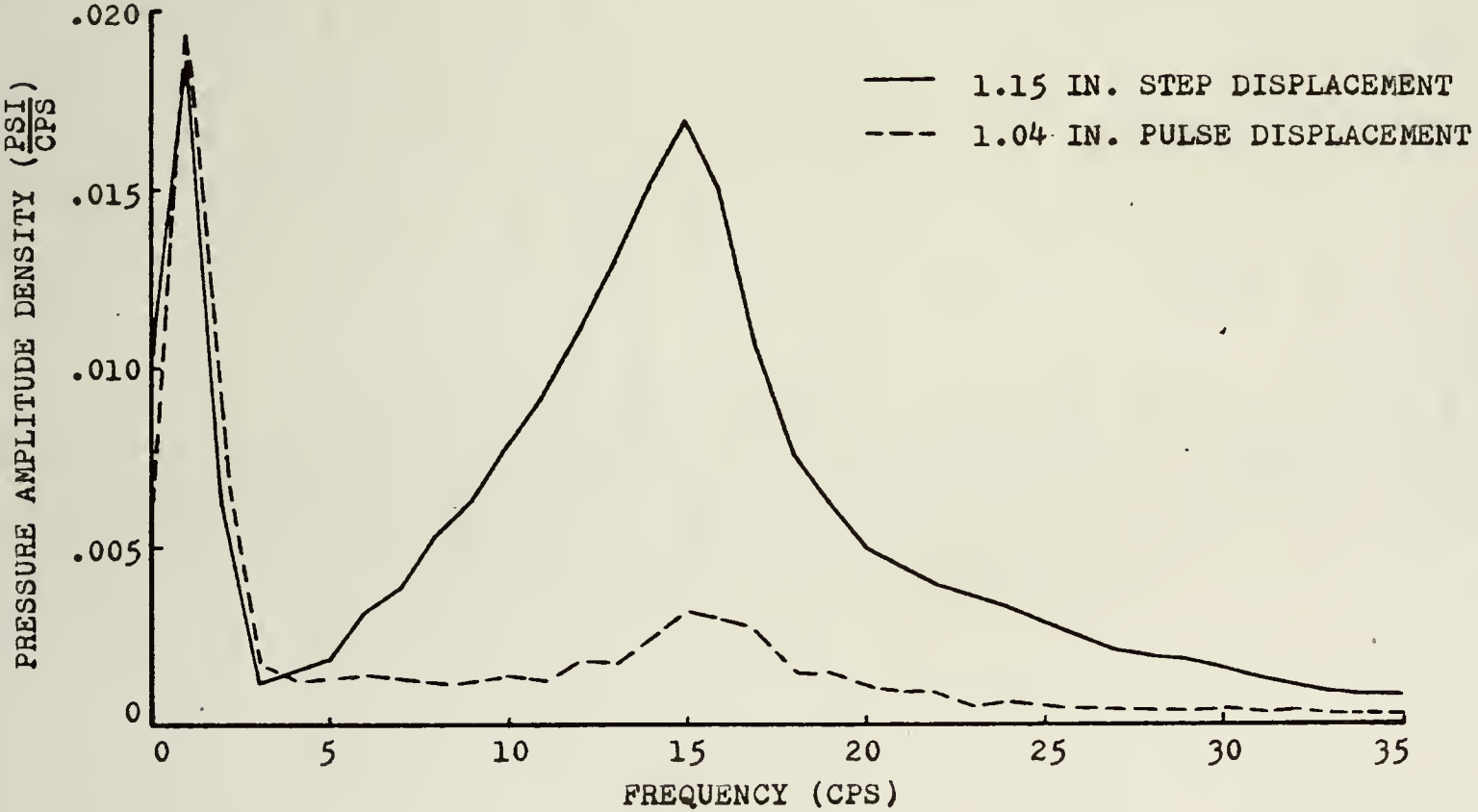


FIGURE 21. FREQUENCY DOMAIN CHARACTERISTICS OF PRESSURE FLUCTUATIONS RESULTING FROM A STEP AND A PULSE DISPLACEMENT INPUT

the air flow through the orifice in the pressure measuring system. As a consequence, the Fourier transform at zero frequency will be a finite value for the pressure, but an infinite value for the force history resulting from a step displacement input. This gives rise to an F/P ratio of infinity at zero frequency. If the step displacement input is to be used, a procedure must be developed for including the effect of the change in the static force, if accurate results are to be obtained. For all frequencies higher than 3 cps a step displacement with a high drop height should be used when the ratio F/P is estimated.

If the amplitude density ordinates of F, P and X are large for a specific frequency, the F/P and F/X calibration ratios for this frequency can be calculated with high accuracy. By comparing the frequency domain descriptions of F, P and X, it is possible to show in what frequency ranges a valid F/P and F/X calibration ratio can be obtained, and also the frequency ranges where the accuracy of F/P and F/X might be questioned. Thus, it is of interest to see the frequency domain characteristics of F resulting from different transient displacement excitations. (See Figures 22 and 23.)

Before proceeding further, an investigation will be made of the minimum X, P and F amplitude density values that can be accepted to give accurate F/X and F/P calibration ratios.

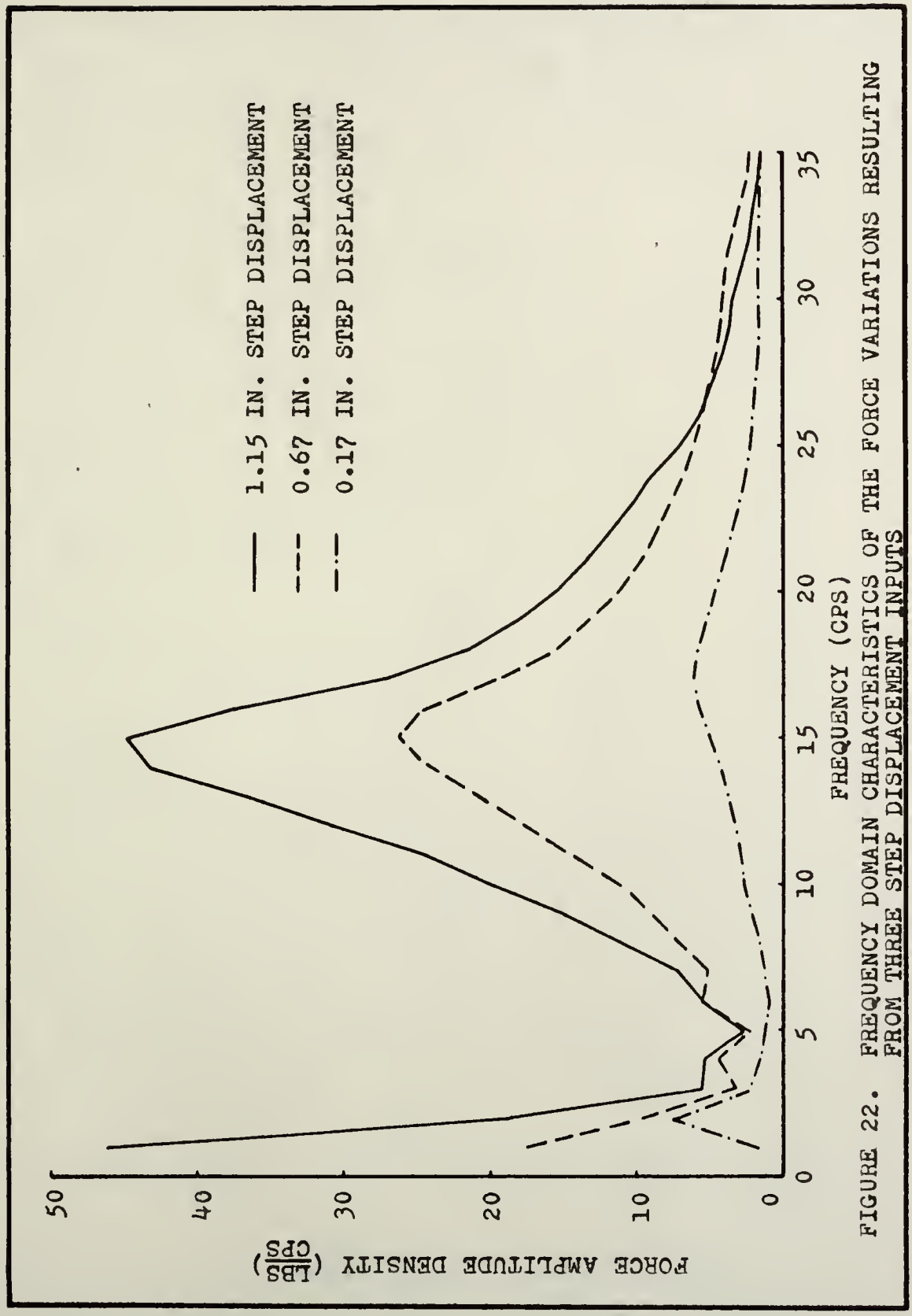


FIGURE 22. FREQUENCY DOMAIN CHARACTERISTICS OF THE FORCE VARIATIONS RESULTING FROM THREE STEP DISPLACEMENT INPUTS

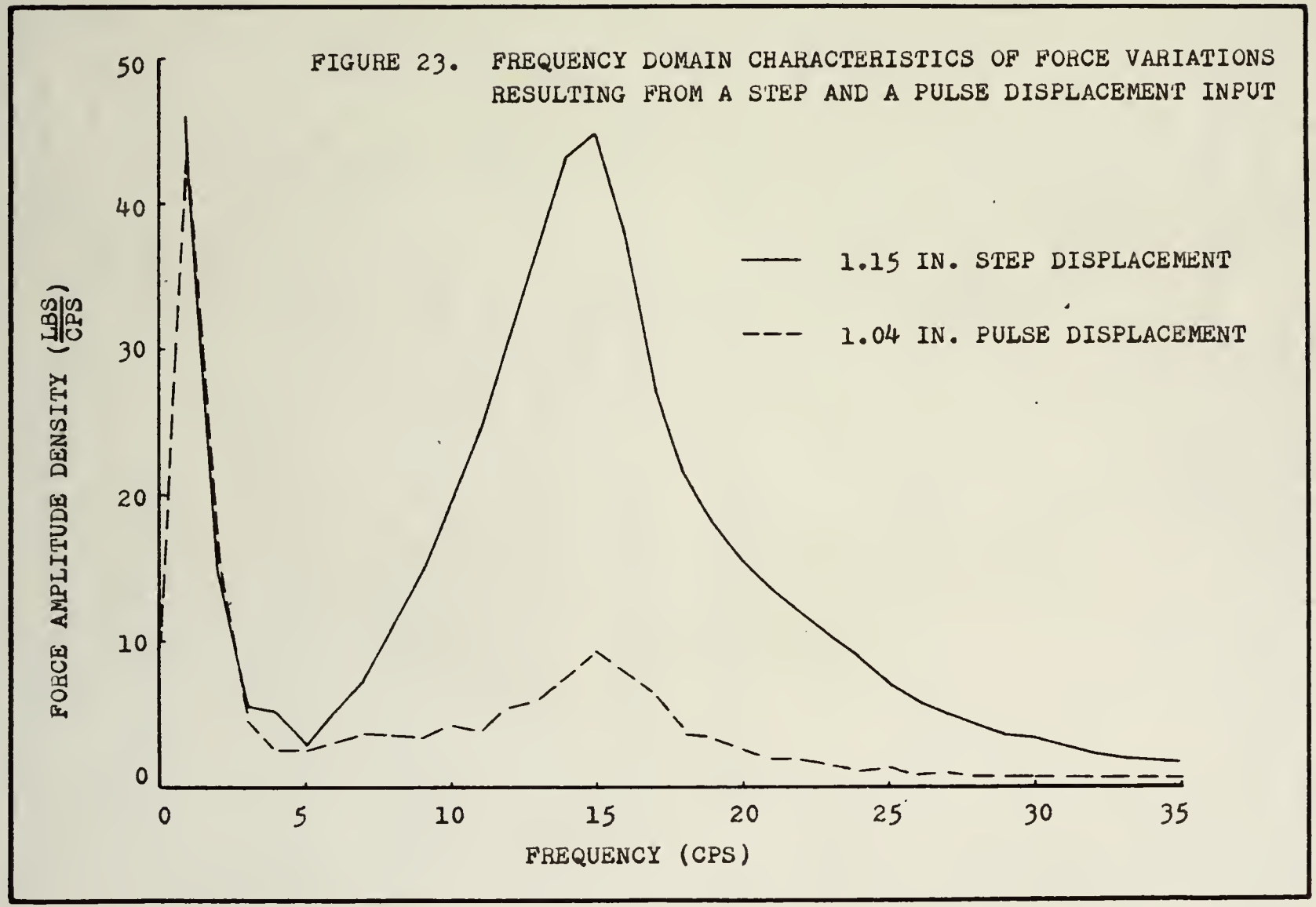


FIGURE 23. FREQUENCY DOMAIN CHARACTERISTICS OF FORCE VARIATIONS RESULTING FROM A STEP AND A PULSE DISPLACEMENT INPUT

In Chapter 3 it was stated that Dreifke[3] gives the following criteria, allowing for about 25% maximum error in the experimental frequency response:

$$|F_1(f)| > 8e$$

$$|F_2(f)| > 8e$$

$$f_{max} < \frac{1}{4\,\Delta t}$$

$$\Delta t \leq \frac{T_1}{10}$$

where

$|F_1(f)|$ = amplitude density of the input at frequency f

$|F_2(f)|$ = amplitude density of the output at frequency f

e = estimated ordinate reading error

f_{max} = maximum frequency for which the amplitude density should be estimated

Δt = time interval (seconds) for reading of data

T_1 = smallest characteristic time of interest in the system

Dreifke[3] used the trapezoidal approximation for numerical integration when the amplitude density values were determined. The frequency characteristics given in

this chapter were obtained by performing a numerical integration using Simpson's rule. Since Simpson's rule gives more accurate results than the trapezoidal approximation, it must be expected that the limitation imposed on $|F_1(f)|$ and $|F_2(f)|$ are conservative. However, the criteria can serve as a rough guide to indicate in what frequency ranges valid F/X and F/P ratios can be obtained.

The records obtained from the calibration tests were read with a spacing of 0.5 mm, and the chart speed was 312 mm/sec. This gives:

$$\Delta t = \frac{0.5}{312} = 0.001602 \text{ seconds.}$$

Then

$$f_{max} = \frac{1.0}{4(0.001602)} = 156 \text{ cps}$$

or

$$T_1 \geq 10(0.001602) = 0.01602 \text{ seconds.}$$

The records were very carefully read, and an estimate of the reading error is difficult to give. However, it might be interesting to find the minimum amplitude density values corresponding to a reading error of $\pm$ 0.02 mm when the 1.15 inch step displacement is used as the transient input.

The calibration constants in this case are:

x calibration factor = 0.0384 inches/mm

F calibration factor = 37.3 lbs/mm

P calibration factor = 0.0165 psi/mm.

Then

$$|X(f)|_{min} = 8(0.0384)(0.020) = 0.00615$$

$$|F(f)|_{min} = 8(37.3)(0.020) = 5.96$$

$$|P(f)|_{min} = 8(0.0165)(0.020) = 0.00264$$

Since the pulse displacement gave high amplitude density values in the frequency range 0-3 cps for X, F and P, it must be assumed that the F/P and F/X calibration ratios in this frequency range can be estimated with high accuracy. Bearing in mind that $\pm$ 0.02 mm is somewhat of an arbitrary reading error, the frequency ranges where the amplitude density values can be used to determine valid calibration ratios will be approximately as follows:

$|X(f)|$ in the 0-16 cps frequency range. (See Figures 18 and 19.)

$|P(f)|$ in the 0-3 cps and 6-25 cps frequency ranges. (See Figures 20 and 21.)

$|F(f)|$ in the 0-3 cps and 6-26 cps frequency ranges. (See Figures 22 and 23.)

From the above, the F/X calibration ratio can be

shown to be valid in the 0-3 cps and 6-16 cps frequency ranges. The F/P calibration ratio will be valid in the 0-3 cps and 6-25 cps frequency ranges, if the 1.04 inch pulse displacement and the 1.15 inch step displacements are used as the transient excitation inputs.

The theoretical analysis in Chapter 3 indicated that a multiple pulse displacement with period of T seconds would give a high input displacement density at a frequency $f_1 = \frac{1}{T}$. Since the frequency amplitude for the step and pulse inputs in the higher frequency range is fairly low, a multiple pulse input could be used to give a more desirable frequency characteristic of the input displacement in this frequency range.

A practical method for accomplishing a multiple pulse displacement input to the wheel of a vehicle would be to let the drop beam, used in the transient calibration tests, fall on a simply supported beam. The supported beam will then act as a spring and introduce an additional frequency of vibration to the drop beam and to the wheel of a vehicle. By changing the distance between the supports, the stiffness of the supporting beam could be changed and thus the frequency of vibration would be changed. An illustration of the experimental set-up is shown in Figure 24.

Figure 25 shows different displacement inputs to a wheel of a passenger car using the supporting beam method.

A comparison between the frequency characteristics

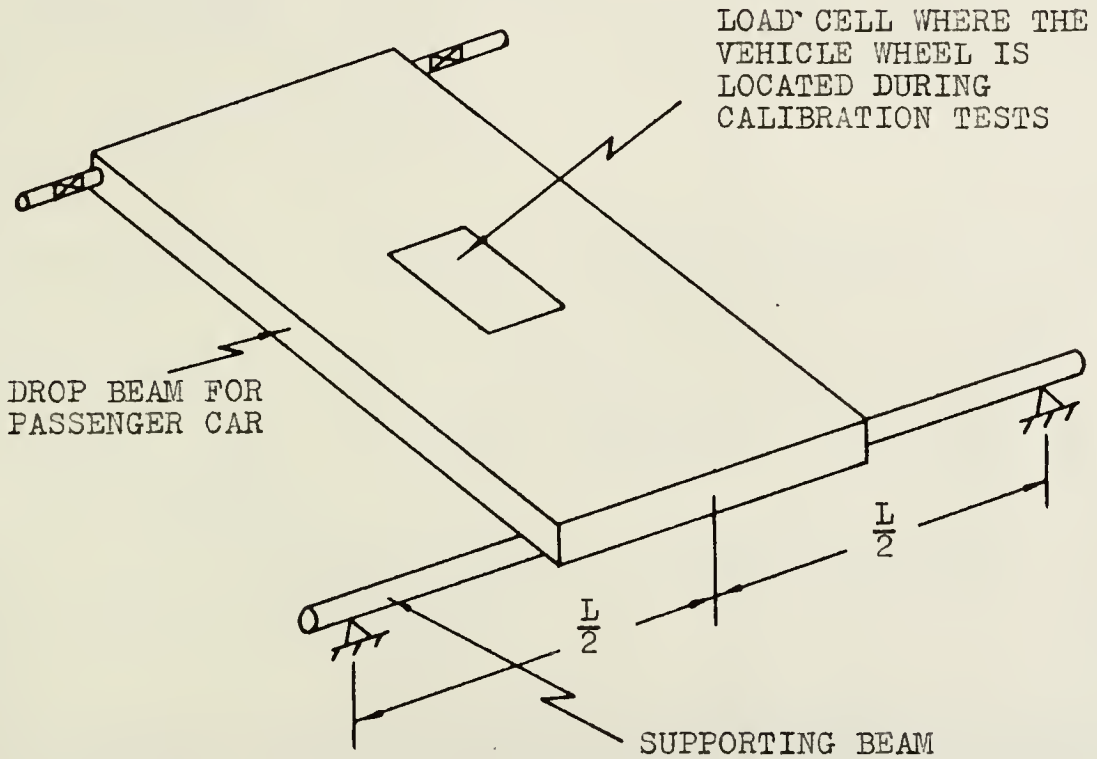
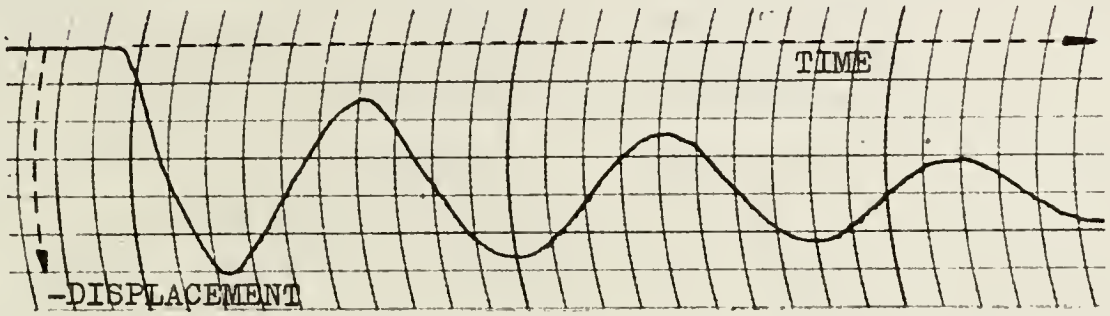
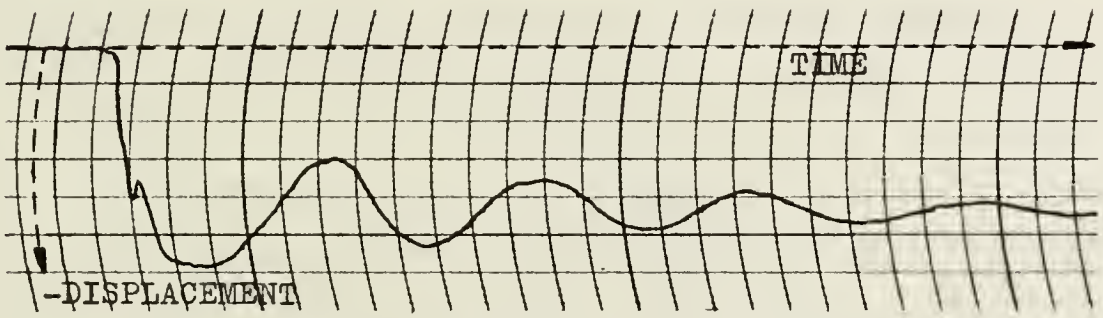

LOAD CELL WHERE THE VEHICLE WHEEL IS LOCATED DURING CALIBRATION TESTS
DROP BEAM FOR PASSENGER CAR
L/2
L/2
SUPPORTING BEAM

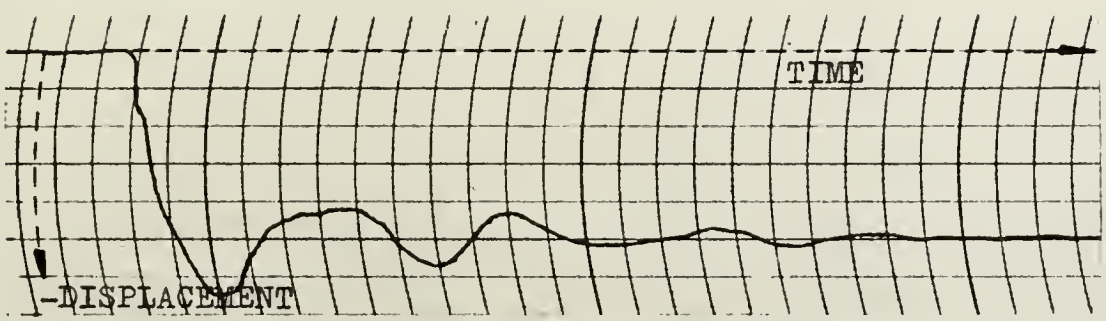
FIGURE 24. ILLUSTRATION OF THE SUPPORTING BEAM SET-UP



DISPLACEMENT RECORD WHEN THE DISTANCE BETWEEN THE SUPPORTS WAS 116 INCHES



DISPLACEMENT RECORD WHEN THE DISTANCE BETWEEN THE SUPPORTS WAS 90 INCHES



DISPLACEMENT RECORD WHEN THE DISTANCE BETWEEN THE SUPPORTS WAS 80 INCHES

FIGURE 25. DISPLACEMENT RECORDS OBTAINED BY USING THE SUPPORTING BEAM METHOD

obtained using this method and an ordinary 0.67 inch step displacement input is given in Figure 26.

It can be seen that although a fairly large increase in the input displacement density was obtained for 8 cps, the frequency amplitude for the higher frequency range was not improved substantially. This can be explained by the fact that the system vibrated almost exclusively at the resonant frequency as seen in Figure 25, and the sharply tuned nature of this system virtually excluded the higher frequencies that may have been excited.

As a result of this investigation it does not appear possible to obtain large input amplitude densities over all frequency ranges that are of interest by using the simple exciting techniques (step, pulse) that have been considered to date.

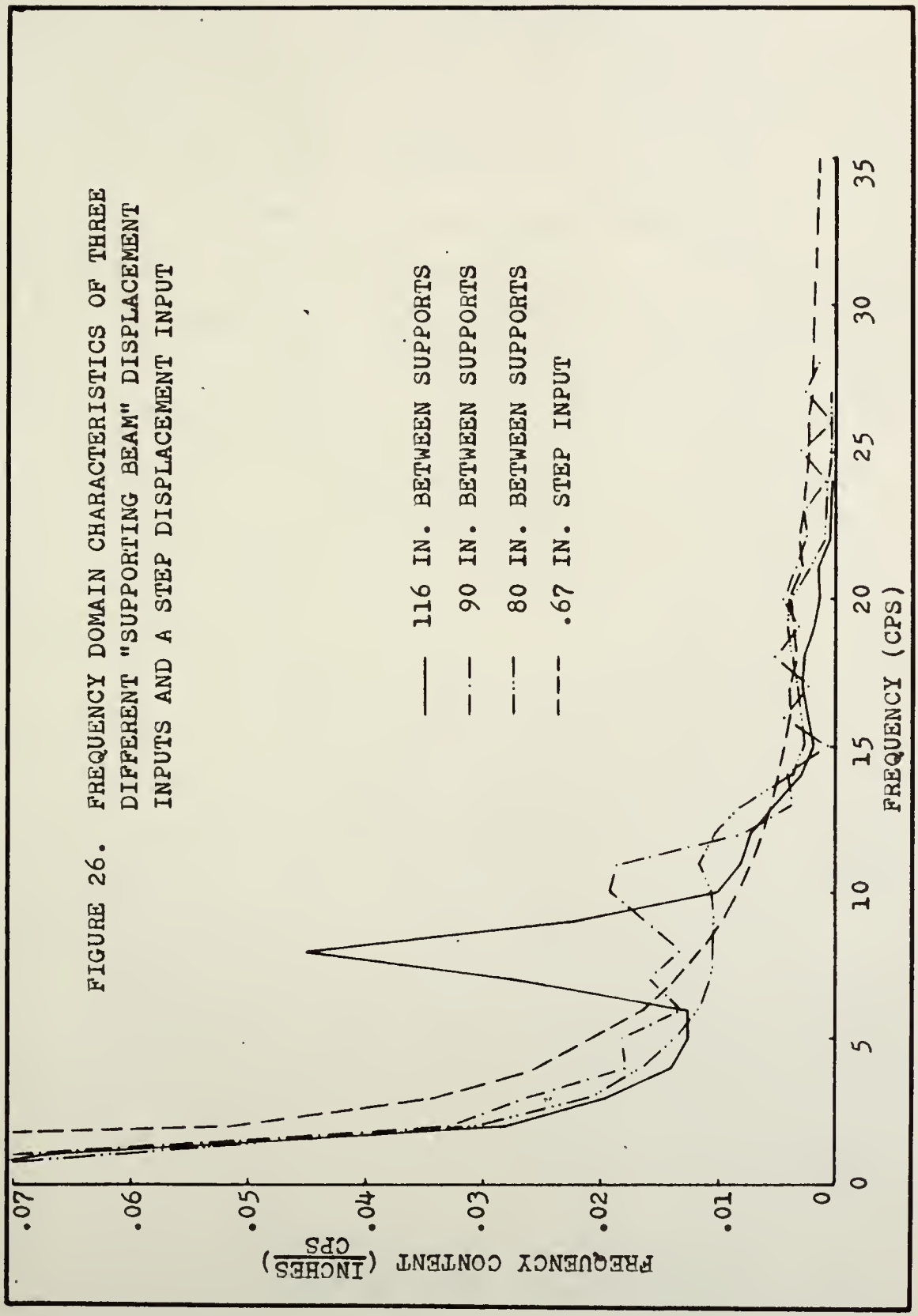

FREQUENCY DOMAIN CHARACTERISTICS OF THREE
DIFFERENT "SUPPORTING BEAM" DISPLACEMENT
INPUTS AND A STEP DISPLACEMENT INPUT
116 IN. BETWEEN SUPPORTS
90 IN. BETWEEN SUPPORTS
80 IN. BETWEEN SUPPORTS
.67 IN. STEP INPUT
FREQUENCY CONTENT (INCHES/CPS)
FREQUENCY (CPS)
.07
.06
.05
.04
.03
.02
.01
0
0
5
10
15
20
25
30
35

FIGURE 26.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The Fourier transform analysis of the step displacement inputs used to determine the dynamic characteristics of vehicles showed that the input displacement density ordinate in the frequency domain was rapidly decreasing as the frequency was increased. A similar frequency domain characteristic was evident for the pulse displacement inputs.

The main purpose of this investigation was to determine the extent to which the "shape" of the present transient excitation inputs (pulse and step) could be changed to increase the input displacement amplitude in the higher frequency range. By doing so, a higher accuracy of the calculated F/P and F/X ratios could be expected.

A study of the frequency regions where a valid F/P and F/X calibration ratio could be obtained by the present transient displacements was also performed. In addition, the possibility of increasing the input displacement density for any specific frequency by using the "supporting beam method" was investigated.

As a result of the investigations conducted during

this study the following observations were made:

1. The input displacement amplitude of a step displacement can be increased by using a greater drop height. However, the physical limitations of the force recording equipment limited the maximum drop height to be about 1.15 inches.

2. The pulse displacement input can produce a more desired frequency domain characteristic if the time duration of the pulse is reduced and the amplitude of the pulse is increased. Since the present method of shaping the pulse depends on manual dexterity, the time duration of the pulse cannot be very short. As a result the input displacement density of the pulse is low for frequencies greater than 3 cps.

3. When the F/P and F/X calibration ratios are evaluated, a pulse displacement should be used for the input in the 0-3 cps frequency range. For frequencies greater than 3 cps, a step displacement input with high drop height should be used.

4. The results obtained by the supporting beam method, indicated that this procedure is not suitable for evaluating the F/P and F/X ratios in the higher frequency range.

5. The present step or pulse displacement inputs

do not have suitable frequency domain characteristics for supplying large excitations to a passenger vehicle over the entire frequency range of 0-35 cps.

6. However, power spectrum analysis of the pressure measurements shows that the most important frequency ranges are those including wheel hop and body motion frequencies. Since the present calibration procedure gives a valid F/P calibration ratio in these frequency ranges for passenger vehicles, a valid force power spectrum can be obtained from a pressure power spectrum by applying this calibration ratio.

It is believed that the transient excitation method is a useful engineering aid for determining the desired vehicle characteristics. To extend the usefulness of the present work, investigations in the following areas are suggested.

1. A device for applying a pulse input to the wheel of the truck should be designed. As mentioned in Chapter 1, provisions for doing this have already been built into the drop beam for the truck. The pit in which the drop beam is mounted is already large enough to accommodate the additional equipment. An investigation has to be performed, however, to determine the nature of the pulse that should be used, and whether or not it might be desirable to use a multiple pulse.

2. A further study of the nonlinear behavior of the suspension system of vehicles, and how this effects the determination of the F/P and F/X calibration ratios, is desired.

3. The present F/X and F/P calibration ratios are obtained when the vehicle is not moving. Since the angular velocity of the wheels when a car is running on a highway will introduce inertia forces in the tires, the actual F/X and F/P calibration ratios during highway driving will differ to a certain degree from the present calibration ratios. A study of this matter is desirable.

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BIBLIOGRAPHY

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